Analytical prediction model for fatigue crack growth in Fibre Metal Laminates with MSD scenario

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Abstract

This paper presents a theoretical and experimental study on Multiple-site Damage (MSD) crack growth behaviour in Fibre Metal Laminates (FMLs). The prediction model is developed based on a simplified analysis of the effects of load redistribution on a single crack in FMLs containing multiple cracks. Test results show that the crack growth accelerates as cracks grow towards each other. The tests also show non-symmetric crack growth behaviour and non-symmetric interfacial delamination propagation in case of multiple cracks. The prediction model successfully captures the crack growth acceleration and non-symmetric growth behaviour.

Keywords: MSD, fibre metal laminates, load redistribution mechanism, non-symmetric crack growth

1. Introduction

FMLs are a family of hybrid laminates made of alternating composite layers and thin metal sheets [1]. The FML concept is evolved out of bonded metal laminate structures by adding fatigue resistant fibres in such laminates, to enhance the crack growth resistance of the metal layers and allow a larger critical damage size. These merits of FML materials enable longer inspection intervals and application of less sophisticated inspection techniques to ensure structural integrity, which is very desirable in the context of the damage tolerance philosophy used in the aerospace sector [2].

An identified deficiency of the damage tolerance philosophy, which relies on inspections, is its compatibility with an unbounded structural life. It has been realised that the damage tolerance philosophy is not enough to secure flight safety for an indefinite life as widespread fatigue damage (WFD) within one structural element can occur over time. In 2010, the airworthiness regulations were revised to include the concept of Limit of Validity (LOV): a period of structural life prior to which WFD will not occur [3, 4]. The LOV concept places limits on the damage tolerance philosophy with the intention to combat WFD failure resulting from crack growth at multiple sites, i.e., the MSD scenario.

Consequently it is crucial to examine the MSD crack growth behaviour in FML materials, even though they are very successful in the context of the damage tolerance philosophy. Several robust damage tolerance analyses of isolated crack growth in FMLs can be found in open literature, including an analytical model for constant amplitude loading by Alderliesten [5, 6] which has been refined for variable amplitude loading [7–9], and for part-through crack configurations [10]. However, these well established damage tolerance prediction models fail to characterise the crack growth behaviour in FMLs containing multiple fatigue cracks and accompanying delamination shapes. Extending these models for the MSD problem is also problematic and cumbersome due to the load redistribution mechanism in FMLs, resulting from the simultaneous presence of multiple cracks [2, 11].

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Hybrid FMLs exhibit different MSD crack growth behaviour compared to monolithic metal sheets, as shown in Fig. 1. Firstly, the growth behaviour of a single crack in an FML is very stable over much longer crack growth life than in a monolithic metallic panel (see Fig. 1), which is attributed to the fatigue resistant fibres remaining intact in the wake of fatigue cracks in the metal layers and acting as a second load path. Then the presence of other cracks in the FML causes local reductions in geometric stiffness at each crack location. As a result, loads redistribute from the adjacent cracks to the single crack. This load redistribution mechanism is present for the whole crack growth life and accelerates the crack growth rate, resulting in a significant life reduction. Whereas in metallic panels the load redistribution mechanism makes only a tiny difference in terms of the crack growth life due to the rapid growth behaviour [12–16]. Therefore, the load redistribution mechanism becomes essential in modelling crack growth behaviour in FMLs containing MSD cracks [2, 11]. A precursor model that analyses the crack growth behaviour in FMLs containing discretely notched layers has been developed by the present authors, with the intention to develop a simplified prediction methodology for modelling the effects of load redistribution on a single crack in an FMLs containing an MSD scenario [2].

The objective of this paper is to integrate two analytical models proposed by the present authors to form another prediction model for analysing the MSD crack growth behaviour in FMLs. Based upon the precursor model [2], this paper analyses the load redistribution mechanism as a result of simultaneous presence of multiple cracks in FMLs in a simplified manner. Additionally, the analytical model which addresses non-symmetric crack tip growth and delamination growth in FMLs [17] will be integrated to simultaneously analyse the asymmetric growth behaviour of multiple cracks. A systematical implementation of the two models will be provided in this paper. The validation is completed by comparing prediction results against the test data of three specimens containing MSD cracks.

2. Background: State of the art in predicting crack growth behaviour for FMLs

2.1. Fatigue crack growth model for FMLs

Fatigue crack propagation in FMLs consists of crack growth in the metal layers and delamination growth at the metal/composite interfaces. As has been stated in the preceding section, the fatigue resistant fibres
remain intact in the wake of fatigue cracks in the metal layers and bridge the cracks, leading to load transfer from the cracked metal layers to the bridging fibres. This load transfer reduces the stresses experienced at the crack tip in the metal layers, and as a result, the stress intensity factor at the crack tip is reduced. Moreover, the load transfer introduces cyclic shear stresses at the interfaces between metal layers and fibre layers, resulting in delamination growth at the interfaces.

Alderliesten has developed an analytical model to calculate the coupled crack growth and delamination propagation using linear elastic fracture mechanics (LEFM) [5, 6]. According to him, the stress intensity factor at the crack tip which is used to characterise crack growth in the metal layers can be decomposed into two components as expressed in Eq. 1. The first term, $K_{ff}$, refers to the stress intensity factor due to stresses in the metal layers due to far field applied load. The second term, $K_{br}$, refers to the stress intensity factor due to the bridging mechanism, which should be superposed upon the first term to derive the total stress intensity factor at the crack tip.

$$K_{total} = K_{ff} + K_{br}$$ (1)

Whereas the strain energy release rate $G$ is applied to characterise the delamination propagation due to the complexity in calculating $K$ at the interface between two dissimilar materials. As expressed in Eq. 2, $G$ varies against $x$ along the crack. Alderliesten subdivides the delamination shape into bar elements perpendicular to the crack and assumes that the delamination grows only in this direction.

$$G = \frac{n_f t_f}{2 E_f} \left( \frac{n_m t_m E_m}{n_m t_m E_m + n_f t_f E_f} \right) (S_f(x) + S_{br}(x))^2$$ (2)

It is noteworthy that both $K_{total}$ and $G$ depend on the bridging stress distribution $S_{br}(x)$ along the delamination frontier [5, 6]. Alderliesten calculates $S_{br}(x)$ by implementing the displacement compatibility between the crack opening and the fibre deformation over the delaminated length at each bar element simultaneously. The crack opening in the metal layers (as a result of crack opening due to the far field load, $V_{ff}(x)$, and crack closing due to the bridging stresses, $V_{br}(x)$, must be identical to the elongation $\delta_f(x)$ and shear deformation $\delta_{pp}(x)$ of the fibre layers.

$$V_{ff}(x) - V_{br}(x) = \delta_{pp}(x) + \delta_f(x)$$ (3)

Eq. 3 can only be solved numerically with a square matrix whose size is identical to the number of bar elements in the delamination shape [5, 6].

2.2. Challenges in extending the model for MSD scenario

The description of the crack growth model for FMLs in the preceding subsection highlights the significance of calculating the crack opening in the cracked metal layers and using a square matrix to implement the displacement compatibility in calculation of the bridging stress distribution. In order to simultaneously solve all the crack states in an FML containing MSD scenario with the Alderliesten model, one faces several challenges. To explain these challenges, an example of MSD scenario in an FML is illustrated in Fig. 2.
The first challenge is to calculate the crack opening displacement at a specific bar element. Crack opening displacements for evenly spaced cracks can be derived [18]. However, the crack opening displacements of non-evenly spaced cracks, which frequently occur in reality, is very challenging to derive. In addition, the crack closing term, $V_{cr}$, in Eq. 3 at the bar element $i$ is not only affected by the bridging stresses over Crack 2, but also by the the bridging stresses over Crack 1 and Crack 3. Therefore these cracks are coupled in terms of the bridging mechanism, which makes the calculation challenging as well.

Secondly, it is computationally inefficient. Each crack has a matrix to be solved and these cracks are coupled with each other as explained above. A very large matrix then has to be constructed in order to solve the bridging stress distributions for all the cracks. The size of the matrix would depend on the number of the cracks and the length of each crack.

These two challenges make the use of the Alderliesten model for MSD scenario cumbersome. Another limitation of the Alderliesten model is that it cannot predict non-symmetric growth behaviour in FMLs [17]. However, as shown in Fig. 2, a non-symmetric crack configuration can arise in case of MSD scenario as a result of crack interaction. This non-symmetry comprises crack tip asymmetry and delamination asymmetry, which has to be dealt with [17].

2.3. Alternative methodology for predicting MSD crack growth

In response to the challenges and limitation discussed above, an alternative methodology has been proposed by the present authors to predict MSD crack growth behaviour in FMLs. This alternative has two essential elements. The first one is to solve each crack state by modelling the effects of other cracks as local reductions in geometric stiffness [2]. The second one is to predict the non-symmetric growth behaviour [17]. These two elements are briefly summarised here and the implementation of these elements for an MSD scenario is given in the next section.

The effects of other cracks on a single crack in an FML can be separated into crack tip interaction effects and load redistribution effects. Crack tip interaction effects are attributed to the interaction of the stress singularities (plasticities) in front of the two crack tips when they are in the vicinity of each other, which occurs for a small portion of fatigue crack growth life before the link-up of cracks. Load redistribution effects are attributed to the reductions in geometric stiffness caused by the presence of other cracks, which is present over the whole fatigue crack growth life. The nature of slow crack growth behaviour in FMLs leads to modelling load redistribution effects being the key factor for predicting MSD crack growth behaviour since the cumulative effects of load redistribution can result in significant reduction in fatigue crack growth life (see Fig. 1) [11]. While the crack tip interaction effects are neglected since the growth rates of two nearby crack tips are so rapid that the error in estimated fatigue life by ignoring these effects is negligible.

This paper adopts the precursor model in [2] to estimate the load redistribution effects in a simplified manner. Other cracks are represented as removals of metal strips when analysing the crack state of a single crack. This representation mimics the reduction in geometric stiffness caused by the crack in the metal layers. The effects of the stiffness reduction is estimated by applying the isostrain condition between the representations of the cracks and the surrounding laminate materials [2]. The reduced stiffness of the laminate at the crack location in front of the single crack causes decrease in the stress transferred by the laminate as a result of isostrain condition, which could lead to load transfer from the crack to the single crack, exacerbating the stress state of the single crack.

The strain distribution, needed for implementing the isostrain condition, is derived from the Westergaard stress distribution in front of the crack tip [2]. In order to model the non-symmetric growth behaviour in FMLs, two different Westergaard stress distributions are assumed in front of the crack tips of a single crack instead of using the identical Westergaard stress distribution for two crack tips [17]. The non-symmetric crack opening displacement of the single crack arising from the two Westergaard stress functions can also be calculated and used to implement the displacement compatibility expressed in Eq. 3. A more generic algorithm for calculating bridging stress distribution proposed in [17] can be used to account for the effects of non-symmetric delamination shapes for the single crack.

With these two elements, the single crack state can be solved without knowing all the crack opening displacements of other cracks, and a relatively small matrix is solved each step with the calculated non-
symmetric crack opening displacement of the single crack. The implementation of the elements will be provided in next section. This process can be iterated until all crack states have been calculated.

3. Model integration and implementation

3.1. Model integration

Two analytical models have been developed with the intention to address the load redistribution mechanism, and non-symmetric crack growth and delamination propagation behaviour in FMLs containing MSD scenario respectively. A systematic integration and implementation of these two models enables analysis of the state of a single crack at each step.

Consider a generic MSD crack configuration illustrated in Fig. 3. Crack $i$ is the single crack to be analysed and the other cracks are idealised as removals of metal strips. The numbering of other cracks is illustrated in Fig. 3(a).

In front of the crack tips of the single crack $i$, two Westergaard stress distributions should be assumed in order to account for the non-symmetry effects [17]. The load redistribution from the other cracks to the single crack is evaluated by implementing the isostrain condition between the representations of the cracks

![Illustration of stress distributions](image1)

![Illustration of balanced equivalent loads](image2)

Figure 3: Illustration of modelling load redistribution and non-symmetry effects
and surrounding laminate material. Consequently, the strain distributions derived from the Westergaard stress distributions in the laminate in front of two crack tips are continuous.

For crack tip $i_1$, the strain distribution in front is:

$$
\varepsilon_{yy,i_1} = \varepsilon_{i_1}/\sqrt{1 - (a_{i_1}/x_{i_1})^2}
$$

(4)

with the subscript $i_1$ denoting that every variable in Eq. 4 is associated with crack tip $i_1$. $\varepsilon_{i_1}$ is an unknown parameter. $x_{i_1}$ starts from the location of the maximum crack opening displacement and the crack length $a_{i_1}$ is also measured from the maximum crack opening location to the crack tip (see Fig. 3(a)) [17, 19]. This maximum crack opening displacement location is a parameter to be determined.

For crack tip $i_2$, the strain distribution in front is expressed as:

$$
\varepsilon_{yy,i_2} = \varepsilon_{i_2}/\sqrt{1 - (a_{i_2}/x_{i_2})^2}
$$

(5)

with the subscript $i_2$ denotes that every variable in Eq. 5 is associated with crack tip $i_2$. $\varepsilon_{i_2}$ is an unknown parameter as well. $x_{i_2}$ is another local coordinates starting from the location of the maximum crack opening displacement and the crack length $a_{i_2}$ another half crack length.

It is worth noting that two half crack lengths $a_{i_1}$ and $a_{i_2}$ are defined instead of using an equal half crack length, $a$, in Eqs. 4 and 5. The definition of two different half crack lengths is attributed to the fact that the Westergaard stress distribution artificially defines a half crack length with the maximum crack opening at the root of the half crack length. In order to obtain a continuous crack opening contour described by two different Westergaard stress distributions, two half crack lengths are defined and the calculated maximum crack opening displacements have to be identical [17].

The stiffness variations in front of the crack tips of the single crack lead to steps in the stress distributions, see Fig. 3(a). At the uncracked parts between crack tips, both the metal layers and fibre layers transfer the load, whereas only fibres at the locations of other cracks transfer the load. In consequence the stress distributions in front of the two crack tips are described through multiplying the strain distributions in Eqs. 4 and 5 by the stiffness of each part in front of the crack tips respectively.

$$
\sigma_{yy,i_1} = \begin{cases}
E_{\text{lam}} \cdot \varepsilon_{i_1}/\sqrt{1 - (a_{i_1}/x_{i_1})^2} = \sigma_{i_1}/\sqrt{1 - (a_{i_1}/x_{i_1})^2} & \text{for uncracked parts} \\
E_{\text{fibre}} \cdot \varepsilon_{i_1}/\sqrt{1 - (a_{i_1}/x_{i_1})^2} = E_{\text{fibre}}/E_{\text{lam}} \cdot \sigma_{i_1}/\sqrt{1 - (a_{i_1}/x_{i_1})^2} & \text{for cracked parts}
\end{cases}
$$

(6)

$$
\sigma_{yy,i_2} = \begin{cases}
E_{\text{lam}} \cdot \varepsilon_{i_2}/\sqrt{1 - (a_{i_2}/x_{i_2})^2} = \sigma_{i_2}/\sqrt{1 - (a_{i_2}/x_{i_2})^2} & \text{for uncracked parts} \\
E_{\text{fibre}} \cdot \varepsilon_{i_2}/\sqrt{1 - (a_{i_2}/x_{i_2})^2} = E_{\text{fibre}}/E_{\text{lam}} \cdot \sigma_{i_2}/\sqrt{1 - (a_{i_2}/x_{i_2})^2} & \text{for cracked parts}
\end{cases}
$$

(7)

$E_{\text{lam}}$ in the above equations represents the overall stiffness of the laminate and $E_{\text{fibre}}$ represents the stiffness of the fibre layers [2]. The reduced stress distributions at the cracked parts indicate the load redistribution effects caused by the reductions in the geometric stiffness.

In Fig. 3(b), $P_{i_1}$ and $P_{i_2}$ are the integrations of the stress distributions ahead of the crack tips respectively. They are the loads carried by the materials in front of the two crack tips. $P_{i_1}$ and $P_{i_2}$ and the far-field load $P_{ff,i}$ needs to maintain not only load equilibrium but also moment equilibrium [17]:

$$
P_{i_1} + P_{i_2} = P_{ff,i}
$$

(8)

$$
P_{i_1}d_{i_1} = P_{i_2}d_{i_2} + P_{ff,i}d_i
$$

(9)

where $d_{i_1}$, $d_{i_2}$ and $d_i$ are the distances between the locations of respective loads and the symmetric line of the panel. They can be calculated using their geometrical relations with the centroids of the respective stress distributions ($x_{c,1}$, $x_{c,2}$ in Fig. 3(b)) [17].
The model is an integration of the two analytical models proposed in [2, 17]. A system of equations can be derived and solved for all the unknown variables. Detailed calculation of all the unknown variables in the above equations can be found in [2, 17]. Then the stress intensity factors at the two crack tips, $K_{ff,1}$, $K_{ff,2}$ can be determined. The effects of load redistribution and non-symmetry are taken into consideration when deriving these stress intensity factors. Meanwhile, a continuous asymmetric crack opening contour $V_{ff,i}$ for the single crack $i$ can also be calculated. For detailed calculation, one can refer to [2, 17].

In addition, two non-symmetric delamination shapes for the two crack tips also contribute to the non-symmetric growth behaviour of the crack tips. The bridging stress distributions, $S_{br,i}(x)$, in the asymmetric delamination shapes over the non-symmetric crack $i$ with asymmetric crack opening contour can also be calculated with the generic algorithm of implementing the displacement compatibility that is detailed in [17]. Then the stress intensity factors due to the bridging mechanism, $K_{br,1}$, $K_{br,2}$ can be determined. The strain energy release rate $G_i(x)$ for crack $i$ can also be calculated with the bridging stress distribution for crack $i$ known. For detailed calculation, please refer to [17].

Eq. 1 needs to be applied to calculate the total stress intensity factors ($K_{total,1}$, $K_{total,2}$) at the tips. The crack growth rates for the two tips can then be determined. So can the delamination growth rate, when $G_i(x)$ is known.

3.2. Numerical implementation procedure

Fatigue analysis itself is an iterative process. Fig. 4 gives an overview of the iterative process for analysing MSD crack growth in FMLs. The methodology for analysing the crack growth behaviour and delamination propagation behaviour in FMLs with multiple cracks is implemented in a numerical program.

The required input variables are shown in Fig. 4. The material properties, loading parameters and Paris constants are standard inputs of fatigue analysis for FMLs. Apart from these inputs, the damage configuration input should be provided. For the case of an MSD scenario, several cracks and their accompanying delamination shapes are present simultaneously. A clear identification system of individual crack and corresponding delamination shape has to be made. In Fig. 5, a three-crack configuration is illustrated as an example. The cracks are numbered from left to right. For a single crack $i$, the associated crack lengths and delamination shapes are identified with the subscripts shown in Fig. 5. The crack length is measured from the hole centre to the individual crack tip. The crack location $x_i$ is the distance measured from the left boundary to the hole centre of crack $i$. The crack lengths and crack locations should be the damage configuration input. The initial delamination shapes for the crack tips are assumed [5, 6].

For a given MSD crack scenario, the state of a single crack can be calculated with the proposed methodology in the preceding subsection. This calculation has to be repeated in order to calculate each crack state, which is the inner loop enclosed by the dashed box in the analysing process illustrated in Fig. 4. A tangible illustration is given in Fig. 6 for a case of three cracks.

Knowing the total stress intensity factors of each crack tip and the strain energy release rate at the delamination over each crack, the crack growth rates and delamination growth rates can be determined with the empirical Paris relations. After a small crack increment and delamination increment, the state of each new crack configuration must be reanalysed. This iterative process continues until two crack tips link-up or one crack tip grows to the free edge.

4. Experimental testing

One variant of FMLs, Glare, was tested to obtain fatigue test data to validate the model. Each specimen has a three-crack configuration, which is illustrated in Fig. 7. The middle crack is at the specimen centre with other two symmetrically located on both sides. The symmetric condition for the middle crack results in symmetric crack growth behaviour of the crack. Apart from validating the present model, the test data of the middle crack will be compared to the prediction for an isolated central crack to highlight the effects of load redistribution caused by the presence of other two cracks. The outer two cracks depicted in Fig. 7 should possess non-symmetric crack growth behaviour. The multiple cracks interact with each other under fatigue loading.
The used Glare comprised of 2024-T3 aluminium sheets and prepregs made of S-2 glass fibre reinforced FM 94 adhesive. Prepregs with different fibre orientations were stacked together to make a fibre layer, Al
layers and fibre layers were then piled up alternately. Standard Glare3 and Glare4B were used [1], the lay-up of each laminate is given in Table 1. The stacked laminates were cured in an autoclave with curing temperature of 120 °C and pressure of 6 bar. The cured laminate panels were milled into the specimen dimensions given in Fig. 7. Three holes with a diameter of 3 mm were applied to each panel. Two notches were cut on both sides of each hole with a hand saw in order to obtain roughly simultaneous crack growth starting from the tips.

Table 1: Test matrix

<table>
<thead>
<tr>
<th>specimen</th>
<th>Glare grade</th>
<th>Al layer number</th>
<th>fibre layer number</th>
<th>prepreg orientation in each fibre layer</th>
<th>applied load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glare3</td>
<td>3</td>
<td>2</td>
<td>0/90</td>
<td>120 MPa</td>
</tr>
<tr>
<td>2</td>
<td>Glare4B</td>
<td>3</td>
<td>2</td>
<td>90/0/90</td>
<td>100 MPa</td>
</tr>
<tr>
<td>3</td>
<td>Glare4B</td>
<td>4</td>
<td>3</td>
<td>90/0/90</td>
<td>100 MPa</td>
</tr>
</tbody>
</table>

The test matrix comprises 3 panels whose configurations are given in Table 1. Specimen 2 and 3 were under same loading but the metal volume fraction (MVF) of specimen 3 is lower than specimen 2 [1]. Due to the fact that the stiffness of Al is higher than that of S-2 fibres, the presence of cracks in the metal layers of an FML with higher MVF results in more reduction in stiffness.

Testing was carried out on an MTS 810 servo hydraulic test frame containing a pin hole clevis and a 250 kN load-cell (model 661. 22D-01). All the fatigue tests were conducted under constant amplitude fatigue loading with stress ratio of $R = 0.05$ and frequency of 10 Hz. The respective maximum applied stresses are given in Table 1. The crack lengths were measured using a monocular microscope with a precision of 0.1 mm while the fatigue test was suspended and maximum stress was applied, the corresponding fatigue life was also recorded. The test resumed after the crack length measurement. The fatigue test stopped when two crack tips linked up.

After fatigue tests, chemical etching was employed to remove the outer aluminium layers to reveal the final delamination shapes. All crack lengths were processed with a 7-point incremental polynomial method recommended in the ASTM E647-00 [20] to obtain the crack growth rate.
5. Results and discussion

5.1. Crack growth behaviour comparison

The crack growth behaviour of all three tested specimens are used to compare against the prediction results. Both crack growth rate comparisons and a-N comparisons are given in Fig. 8, Fig. 9 and Fig. 10 for each specimen respectively.

For each specimen configuration, the predictions for an isolated central crack using the Alderliesten model [5, 6] are also provided as dashed lines. The comparison between the predicted growth rate of the isolated crack and that of the middle crack ($a_{21}$, $a_{22}$) in the case of MSD highlights the increase in the crack growth rate due to the stiffness reductions caused by multiple crack growths. The predicted growth rate without considering the effects of load redistribution for MSD scenario is highly inaccurate and the cumulative error leads to over prediction of the fatigue life by a factor of about 1.8 (see Fig. 8(b), Fig. 9(b), Fig. 10(b)).

In the present prediction methodology, the load redistribution effects are accounted for throughout the entire crack growth period. As can be seen in Figs. 8(a), 9(a) and 10(a), the predicted crack growth rates correspond to the experimental measurements well for low Damage Volume Fraction (DVF). The DVF is proportional to the crack lengths. As the crack lengths increase, the DVF grows from low to high for each FML. The predicted crack growth rates are conservative for high DVF with long crack lengths. The prediction model captures the crack acceleration feature of interacting crack tips in FMLs. However, the idealisation of the cracks in modelling load redistribution leads to over predicted results.

For the major portion of the crack growth and the major portion of the growth life, two crack tips that grow towards each other are not close enough for the stress singularities (plasticities) in front to interact. It is the load redistribution mechanism that increases the crack growth rate. Whereas the effects of plasticity interaction present when one crack tip is in the vicinity of another, resulting in dramatic increase in the crack growth rate, shown in Fig. 9 and Fig. 10. The load redistribution effects are modelled throughout the whole crack growth even in the presence of the plasticity interaction, however the stress singularity interaction effects in the vicinity of two crack tips are ignored.
The calculation of the size of plasticity in front of the crack tip and the stress singularity interaction is beyond the scope of this paper. Eq. 10 is merely used to provide an indication of the plastic zone size $r_p$. The total stress intensity factor $K_{total}$ for $a_{22} = 13$ mm in specimen Glare4B – 4/3 – 0.4 and the yield strength $\sigma_{ys} = 347$ MPa for the used aluminium are substituted into Eq. 10. The obtained plastic zone size is 1.1 mm. Neglecting the plasticity interaction effects therefore leads to underestimation of the crack growth rate only over an estimated crack length of roughly 1 to 2 mm before link-up of the crack tips (see Fig. 10(a)). It is worth noting that this plasticity interaction only occurs over a rather small portion of crack length with rapid growth rate, resulting in a minimum error in fatigue life prediction without considering this in the model.

Figure 8: Comparison between prediction and measurements of Glare3-3/2-0.4 specimen

Figure 9: Comparison between prediction and measurements of Glare4B-3/2-0.4 specimen
The deviations in predicted fatigue crack growth life and measured crack growth life in a-N comparison figures are attributed to the over predicted crack growth rates. The over prediction is a result of the idealisation of adjacent cracks as removal of strips of metal layers when analysing the crack state of a single crack in the prediction model. The idealisation is not a true physical representation of cracks in the proposed model, which introduces more stiffness reduction and thus more load redistribution in modelling than actual cracks do in a laminate. However, the load redistribution is adequately captured by the non-physical representation. The over predicted crack growth rates lead to conservative calculation results. Otherwise the fatigue growth life prediction could be very non-conservative if the load redistribution effects are not considered, illustrated by the predicted life of an isolated crack growing to the length where the link-up occurs for the tested MSD cracks (Fig. 8(b), Fig. 9(b), Fig. 10(b)).

The predicted crack growth life and the measured crack growth life for each specimen are given in Table 2. The MVF which is defined as the ratio between the total thickness of metal layers and the total thickness of the laminate, and the relative error between the prediction and the measurement are also given. The DVF is also proportional to the MVF of FMLs containing the same crack lengths. A consistent trend between the MVF and the relative error can be observed: the relative error is smaller with lower MVF. In other words, the predicted fatigue life is more accurate for an FML with a low DVF than that for an FMLs with a high DVF. This can be attributed to the idealisation in the prediction model. The influence of other cracks on a single crack is estimated by idealising the cracks as negative stiffeners, which enlarges the reduction in stiffness caused by real cracks in the model. The model tends to enhance the enlargement for FMLs with higher MVF, leading to more conservative crack growth rate prediction and thus more error.

![Figure 10: Comparison between prediction and measurements of Glare4B-4/3-0.4 specimen](image)

\[
r_p = \frac{K_{total}^2}{\pi \sigma_{ys}^2}
\]

Table 2: Prediction accuracy vs MVF

<table>
<thead>
<tr>
<th>specimen</th>
<th>Glare grade</th>
<th>MVF</th>
<th>measured life</th>
<th>predicted life</th>
<th>relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Glare3-3/2-0.4</td>
<td>0.69</td>
<td>74000</td>
<td>51600</td>
<td>-30.3%</td>
</tr>
<tr>
<td>2</td>
<td>Glare4B-3/2-0.4</td>
<td>0.60</td>
<td>95500</td>
<td>70800</td>
<td>-25.9%</td>
</tr>
<tr>
<td>3</td>
<td>Glare4B-4/3-0.4</td>
<td>0.57</td>
<td>104000</td>
<td>83400</td>
<td>-19.8%</td>
</tr>
</tbody>
</table>
5.2. Delamination comparison

The proposed model also predicts the individual delamination shape evolution in an FML containing MSD cracks. The comparison between prediction and measurement is made for specimen 1 (see Table 1) in Fig. 11. The delamination measurements were made after etching the outer aluminium layers of specimen 1 away. Due to the symmetry, delaminations of crack tip 1, 2 and 3 are shown only. An very good correlation can be observed in the comparison.

![Delamination comparison](image)

Figure 11: Delamination comparison for specimen 1

Due to the interaction effects of crack tip 2 and crack tip 3, their delamination shapes are larger than the delamination for crack tip 1. The good correlation between calculated and measured non-symmetric delamination shapes for crack tip 1 and crack tip 2 of the single crack indicates high accuracy of the predicted non-symmetry effects by the proposed model.

6. Conclusion

A new prediction methodology for MSD growth in FMLs has been developed based on other two models proposed by the present authors. The proposed model in this paper analyses both crack interaction in terms of load redistribution mechanism and non-symmetric growth behaviour of multiple cracks in FMLs. The load redistribution resulting from the presence of multiple cracks is modelled by idealising the cracks as removals of metal strips and applying the isostrain condition at the crack locations. The non-symmetry effects are modelled by applying two different Westergaard stress distributions in front of two tips of a single crack that is being analysed. These two mechanisms have to be simultaneously solved to obtain the state of a single crack, which is iterated to determine all crack states sequentially.

Based on the analysis and results obtained in this paper, the following remarks can be made:

1. The nature of fatigue crack growth in FMLs leads to the significance of modelling load redistribution in analysing MSD scenarios in FMLs. Neglecting the interaction of stress singularities in the vicinity of two crack tips results in a negligible error in terms of fatigue growth life. However, not considering the load redistribution mechanism for MSD scenario could result in a very non-conservative predicted fatigue life.

2. The non-physical representation of cracks leads to conservative prediction results. However, it adequately captures the crack growth acceleration in the case of MSD cracks in FMLs.

3. The non-physical idealisation of cracks as removal of metal strips in modelling the load redistribution mechanism exaggerates the stiffness reduction caused by the actual cracks. Therefore the predicted results are more accurate for FMLs with lower MVF since the exaggeration is less (Table 2).
4. In addition to the non-symmetric free boundary edges and crack configurations in front of two tips of a crack, the asymmetric delamination shapes for two crack tips also contribute to the non-symmetric crack growth behaviour.

5. The non-symmetric crack lengths and delamination shapes are well predicted by the proposed model.

6. The presence of fatigue resistant fibres in FMLs alleviates the stiffness reduction due to the co-existence of multiple cracks compared to monolithic metals.

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References