Philosophy of Multiple-site Damage Analysis for Fibre Metal Laminate Structures

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Abstract: This paper outlines the proposed approach and current implementation progress for a multiple site damage model for Fibre Metal Laminate structures. The current state of the art in FML crack growth prediction utilizes linear elastic fracture mechanics, displacement compatibility, and superposition in order to characterize the coupled crack/delamination growth behaviour of a fatigue damaged FML. Directly extending this approach to a MSD scenario introduces several challenges which will be outlined and discussed. A computational simplification of idealizing cracks as “negative stiffeners” in order to simplify the estimation of stress intensity factors at single crack locations will be substantiated and discussed.

NOMENCLATURE

- $a$: Crack length
- $a_s$: Saw-cut length
- $K_\infty$: Stress intensity factor due to far-field applied load
- $K_\text{br}$: Stress intensity factor due to bridging load
- $K_{\text{total}}$: Total stress intensity factor at the crack tip
- $K_{\text{red}}$: Stress intensity factor due to applied load and redistributed load
- $n_f$: Number of fibre layers
- $n_m$: Number of metal layers
- $G$: Strain energy release rate
- $E_f$: Young's modulus of fibre layer
- $E_m$: Young's modulus of metal layer
- $t_f$: Thickness of fibre layer
- $t_m$: Thickness metal layer
- $S_f$: Stress in the fibre layers
- $S_{\text{br}}$: Bridging stress
- $j$: Number of interfaces
- $v_c$: Crack opening displacement due to far-field load
- $v_{\text{br}}$: Crack closing component due to bridging load
- $\delta_p$: Elongation of prepreg layer
- $\delta_{pp}$: Shear deformation of prepreg layer
- $da / dN$: Crack growth rate
- $db / dN$: Delamination growth rate
- $\Delta a$: Crack extension
- $\Delta b$: Delamination bar length extension
- $\Delta N$: Applied fatigue cycles

INTRODUCTION

Fibre Metal Laminates (FMLs) are a class of hybrid material and comprise metal layers bonded with composite layers. Their well-known superior fatigue crack resistance and longer critical crack length, which are desirable in the context of damage tolerance design philosophy utilized in the aerospace sector, are attributed to the bridging mechanism offered by intact fatigue resistant fibres in the wake of a fatigue crack in FMLs. See Figure 1. The philosophy of damage tolerance assessment for fatigue crack growth in FMLs has been well established. These damage tolerance analyses for
FMLs mainly focus on predicting the evolution of an isolated crack. As a result, they may be invalid due to the presence of Multiple-site Damage (MSD) cracks in fibre metal laminate structures.

An identified deficiency of damage tolerance design philosophy is its compatibility with an indefinite structural life. The use of this philosophy to ensure structural integrity becomes problematic in the case of MSD scenario within an aging structure. In order to combat the possibility of the failure caused by occurrence of MSD scenario, the airworthiness regulations has been revised in 2010 to include the definition of a Limit of Validity (LOV) placing limits on damage tolerance analyses. It is therefore crucial to examine MSD crack growth behaviour of fibre metal laminates even they possess superior damage tolerant behaviour.

Figure 1. Bridging mechanism.

The models for damage tolerance analyses of FMLs in open literature predict crack propagation in metal layers and extension of delamination at the metal-composites interfaces using classical Linear Elastic Fracture Mechanics (LEFM). Calculating the bridging stress distribution in the intact fibres bridging the fatigue crack is essential for predicting the coupled failure mechanisms in FMLs in these models. However, directly extending these models for simultaneously predicting the crack growth behaviour of MSD cracks in FMLs is cumbersome and computationally inefficient due to the complicated calculation of load redistribution among multiple cracks and corresponding bridging fibres.

This paper describes the philosophy with the aim of proposing a simplified prediction methodology for analysing MSD behaviour in FMLs. This paper explains the challenges of directly applying the models of damage tolerance analysis for FMLs to examine MSD behaviour in FMLs. The flow diagram of the presented philosophy for MSD analysis of FMLs is shown in this paper. The general utilized approach in this proposed philosophy is classical LEFM, in conjunction with the principle of superposition and displacement compatibility. The idealization of crack interaction in terms of load redistribution in FMLs containing MSD scenario will be illustrated.

Additionally, this paper will highlight the importance of predicting behavior of an eccentric crack with two different delamination shapes on both sides of its saw-cut in a FML panel, and the compatibility of this eccentric crack prediction with the MSD growth prediction in FMLs.

NATURE OF FATIGUE IN FMLS

As a result of their hybrid nature, FMLs have different fatigue behaviour compared to monolithic metal. The fatigue resistant fibres which remain intact in the wake of fatigue cracks in metal layers in FMLs act as a second load path and restrain the opening of the cracks. Due to this bridging mechanism, the stress intensity factors are considerably reduced and the crack growth in FMLs is quite slow in comparison with monolithic metal. The bridging mechanism, however, is not effective in the crack initiation phase for FMLs [1]. In addition, the cyclic stress in the metal layer of FMLs is a superposition of the stress induced by the applied load and the curing stress [1, 2]. These two factors result in a quite short initiation life for FMLs.

Figure 2 shows the contributions of the initiation life and crack growth life to the total fatigue life for FMLs and monolithic aluminium. The differences in contributions are typical for FMLs and monolithic metals [1]. As a result of the very fast crack growth in monolithic metal, the crack growth life only accounts for a very small part of its fatigue life. On the contrary, the superior fatigue crack growth resistance of FMLs results in a considerably long crack growth life which covers the main part of the total fatigue life.
The difference in ratio of crack growth life to overall fatigue life for metals and FMLs has implications on the fatigue behaviour of such materials in an MSD scenario. In order to discuss these differences, the influence of MSD cracks will be divided into two categories: load redistribution effects and crack-tip interaction effects. Crack tip interaction effects occur when the proximity of multiple crack tips results in an interaction of the stress singularities in front of the crack tips. For two cracks approaching each other, this tends to result in rapid fatigue propagation and ultimately linkup of the cracks. Load redistribution effects occur due to a loss in component stiffness due to the presence of multiple damages. These effects will be present over the entire range of damage sizes and proximities (i.e.: they are also present when crack-tip interaction effects are present), although they are easier to describe for cracks sufficiently far that crack-tip interaction effects are negligible. In such a case, the crack propagation of one crack will be increased due to the increase in stress in the undamaged region of the component resulting from other damaged regions.

The relative crack growth behaviour of metals and FMLs, with and without MSD, is shown schematically in Figure 3. The relatively rapid crack growth behaviour of the metal implies that cracks in an MSD scenario will grow towards each other relatively quickly resulting in early linkup. Crack linkup will rapidly increase the size of the primary crack resulting in a reduction of fatigue life. The reduction in crack growth life is thus heavily influenced by crack-tip interaction effects. Conversely, the slower fatigue crack growth rate in FMLs combined with the shorter crack initiation life means that a significant portion of the MSD crack growth life in FMLs can occur well before the presence of crack-tip interaction effects. During this time, the cumulative effects of load redistribution resulting from the presence of multiple damages can result in a significant reduction in fatigue life (ΔN).

**DAMAGE TOLERANCE ANALYSIS FOR FMLS**

Since the proposed philosophy is based on the methodologies used in damage tolerance analysis for FMLs, a brief review of the well-established FML crack growth model developed by Alderliesten [7, 8] is given before the proposed philosophy is detailed.

Using the Linear Elastic Fracture Mechanics (LEFM) together with principles of superposition and displacement compatibility, Alderliesten has successfully made the prediction of coupled propagation of cracks in metal layers and delaminations at metal/composites interfaces [7, 8]. Both crack extension and delamination extension are determined...
with LEFM method, while stress intensity factor, \( K_s \), is used to characterize crack extension in metal layers (Eqn. 1) and strain energy release rate, \( G_s \), is applied to estimate the extension of delamination (Eqn. 2). For both crack growth resistance and delamination extension resistance of FMLs, experimentally determined Paris relations are applied [7].

\[
K_{\text{total}} = K_s + K_{br}
\]  
(1)

\[
G = \frac{n_j t_f}{2 E_f} \left( \frac{n_d t_d E_d}{n_d t_d E_d + n_f t_f E_f} \right) \left( S_f + S_{br} \right)
\]  
(2)

The principle of superposition is applied in Eqn.1, i.e. the total stress intensity factor, \( K_{\text{total}} \), is decomposed into \( K_s \) due to the external applied load plus internal residual stress, and \( K_{br} \) due to the stress reduction caused by the bridging fibres in the wake of a fatigue crack. The strain energy release rate, \( G \), is also a function of bridging stress (Eqn. 2). The bridging stress distribution therefore needs to first be determined in order to resolve the crack extension and delamination growth.

The principle of displacement compatibility has been applied by Alderliesten to resolve the bridging stress distribution over the cracked region [7]. Since the fibre layers are bonded to the metal layers at the delamination front, the displacement of the fatigue cracked metal layer should be identical to the deformation of the bridging fibres over the delaminated length. See Eqn. 3. The displacement of cracked metal layer, which is given on the left side of Eqn. 3, is decomposed into opening displacement, \( v_\infty \), due to the external applied load and residual stress and crack closing item, \( v_{br} \), due to the bridging stresses in bridging fibres. The deformation of the bridging fibres over the delaminated length includes elongation of the prepreg layers, \( \delta_f \), and shear deformation of the prepreg layers, \( \delta_{pp} \).

\[
v_\infty (x) - v_{br} (x) = \delta_f (x) + \delta_{pp} (x)
\]  
(3)

As the displacement of the metal layer and the deformation of the bridging fibres vary along the crack flank, the delamination shape is evenly divided into bar elements to implement Eqn. 3 at each bar location for solving the bridging stress. Figure 4 schematically illustrates the division of the delamination shape. Moreover, the crack closing component, \( v_{br} \), at one bar location is not only a function of the bridging stress at the bar element location, but affected by the bridging stress at each bar element. As a result, the displacement compatibility has to be implemented simultaneously for all the bar elements, forming a system of linear equations which can be expressed with matrix notation containing a square matrix whose size is equal to the number of bar elements. Once the bridging stress distribution is calculated, the crack growth rate and delamination growth rate can be determined by using Eqn. 1 and Eqn. 2 together with their Paris relations.

**Figure 4. Bar element division for solving bridging stress distribution.**

### ISSUES OF SIMULTANEOUS PREDICTION OF MULTIPLE COLLINEAR CRACKS

Based on the description in the preceding section, it is essential to calculate the bridging stress distribution by implementing displacement compatibility for determining the crack extension and delamination extension in FMLs.
Extending this approach to simultaneous prediction of crack propagation and delamination growth of multiple collinear cracks (MSD scenario) in FMLs seems to be a natural choice. However, simultaneously predicting all the crack states of the collinear cracks in FMLs has two major issues.

The first issue is to calculate the crack opening displacement (COD) for a row of cracks depending on the configuration. This will be explained by an example depicted in Figure 5 which schematically illustrates a row of 3 cracks in FMLs with the delamination zones divided into bar elements. In order to implement the principle of displacement compatibility at the location of bar element $i$, the crack opening displacement, $v_i$, of Crack 2 is required as an input in Eqn. 3. However, the COD of Crack 2 is difficult to be calculated due to the presence of other two collinear cracks: Crack 1 and Crack 3. Although the COD solutions for some specific MSD configurations are available in literature [10-12], it is a challenging issue to calculate COD for generic MSD configuration.

The second issue is the computational efficiency of simultaneous prediction of the bridging stress at all bar elements over multiple crack spans. As an example illustrated in Figure 5, the closing displacement, $brv_i$, of bar element $i$ is a function of the bridging stresses at all bar elements of Crack 1, Crack 2 and Crack 3. Thus a matrix whose size could be around 3 times larger than that for one crack configuration needs to be constructed for the same reason described in the preceding section. Depending on the number of cracks and the length of each crack of generic MSD configuration in FMLs, the much larger matrix constructed to predict bridging stress distribution results in computational inefficiency.

**ANALYSIS METHODOLOGY FOR FML STRUCTURES CONTAINING MSD**

**Effect of stiffness variation**

The effective stiffness of a structure is related to the material stiffness (Young’s modulus) and geometric stiffness of the structure [13]. The material stiffness of a panel is an inherent property which cannot be changed merely by the presence of a crack. However, the presence of a crack can decrease the geometric stiffness of a structure by reducing the net sectional area, resulting in larger deformation under same loading condition as the remaining material has to carry the entire applied load. Then the crack growth rate increases with the crack length [13]. In case the cracked panel is reinforced by stiffeners which increase the effective stiffness of the cracked panel, the deformation is restrained. The crack growth rate decreases when the crack grow towards an adjacent stiffener as a result of the load redistribution from the cracked skin to the stiffeners [13]. Another example is that the crack growth in FMLs is much slower in the comparison of the fast crack growth of one crack in a monolithic metal panel because the intact bridging fibres partially compensate the geometric stiffness reduction in the metal layers [14].

The reverse case of a cracked panel reinforced by stiffeners is the presence of MSD cracks in a panel. In addition to one single crack in a monolithic metal panel, the presence of collinear adjacent cracks prohibitively decreases the effective stiffness of the panel as no other material could compensate the stiffness reductions. As a result, the crack growth rate of the single metal crack prematurely soars up. By contrast, the effect of the presence of adjacent MSD cracks on the growth behavior of the single crack in FMLs is not as pronounced as in metal. See Figure 3. This is attributed to the compensation made by the intact bridging fibres in the wake of MSD cracks to the stiffness reductions in metal layers. The presence of MSD cracks in FMLs still results in stiffness reductions which lead to load redistribution from the locations of MSD cracks to the single crack. Figure 6 demonstrates the load redistribution from an adjacent crack to the centre crack in FMLs.

Properly estimating the effect of stiffness reduction caused by the presence of MSD cracks on the single crack in FMLs and applying the methods used in damage tolerance analysis for FMLs to resolve the single crack state comprise the simplified analysis philosophy for FMLs with MSD problem.
A precursor model has been developed to predict fatigue crack growth in FMLs panels containing discretely notched layers. The objective of the precursor model is to verify the concept of load redistribution caused by stiffness reduction which is achieved by discretely notched layers [9]. The difference between discretely notched metal layers and fatigue cracked metal layers is the severity of stress concentration/plasticity at the notch edge/crack tip in the crack section. For the development of the model and the test procedure, please refer to [9]. Only Figure 7 from [9] is given to illustrate the effects of different stiffness reductions on the crack growth behaviour in FMLs.

As can be seen in Figure 7, the crack growth rate with discretely notched Al layers is higher than that with notched fibre layers and M (T) specimen. Because the stiffness reduction resulted from notched Al layers is much higher than notched fibre layers, more load transferred from notched area to the crack tip in FMLs. The little stiffness reduction resulted from notched fibre layers leads to limited load redistribution and increase in crack growth rate compared to M (T) specimen [9].

Another feature in Figure 7 is that the possible stress concentration/plasticity interacting zone covers a small portion of total crack length. Most part of the crack growth is affected by the load redistribution mechanism. In addition, the relatively slow crack growth rate resulted from load redistribution and much higher crack growth rate in stress concentration/plasticity interacting zone result in that neglecting the stress concentration/plasticity interacting does not affect the total crack growth life prediction too much. The influence should be acceptable then in terms of total fatigue life.
Analysing eccentric crack growth in FMLs

It is notable that most of MSD cracks are not symmetric, each asymmetric crack has different crack lengths with different delamination shapes on both sides of its saw-cut, as shown in Figure 8. The delamination shapes in Figure 8 can be examined by etching away the outer metal layers of FML specimen after fatigue testing [7, 15]. This eccentric crack phenomenon raises new challenges for the analysis of MSD crack growth in FMLs: asymmetric COD for an eccentric crack and asymmetric bridging stress distribution with respect to the crack centre. Consequently, the solutions applied by Alderliesten [7] to resolve bridging stress distribution for a central crack with symmetric delamination shapes and crack lengths in FMLs are no longer applicable to an eccentric crack in FMLs. New analysis methodology for an eccentric crack in a finite FML panel is needed to be extended to fit in with the MSD growth analysis for FMLs.

Figure 8. MSD cracks in a FML panel.

The symmetric condition of a finite panel is damaged by the presence of an eccentric crack, the COD for the eccentric crack therefore is not symmetric and the stress distributions in front of the crack tips are not the same either. See Figure 9. In open literature [11, 16, 17], only different stress intensity factors for two crack tips of an eccentric crack in a finite metal panel are given. However, the asymmetric COD of an eccentric crack is yet needed to implement the principle of displacement compatibility to calculate the bridging stress distribution in FMLs. The asymmetric COD of an eccentric crack in metal layer result in asymmetric bridging stress distribution and thus different delamination shapes, as schematically illustrated in Figure 9.

Figure 9. Illustration of an eccentric crack in a FML panel.

Two different Westergaard stress distributions, illustrated in Figure 9, can be assumed in front of the two crack tips of an eccentric crack in metal layers in FMLs. If these two Westergaard stress distributions can be calculated, then the asymmetric crack opening displacement for the eccentric crack can be obtained [18]. This method also permits the possibility of extending the model for eccentric crack analysis in FMLs to estimate the load redistribution caused by the presence of MSD cracks, in the same way as described in [9].
The method used by Alderliesten [7] to calculate the displacement closing component, $v_c$, in Eqn 3, for symmetric delamination scenario cannot be used to calculate the bridging stress for asymmetric delamination scenario. Then the Westergaard stress method for two points loading on Page 5.6 in crack analysis Handbook [10, 11] has to be applied to calculate the corresponding closing component in order to implement the principle of displacement compatibility.

Investigating the crack growth behaviour of an eccentric crack in FMLs is a necessary step for analyzing MSD crack growth in FMLs. The analytical model for analyzing eccentric crack growth in FMLs should have the capability to be extended to calculate the effect of load redistribution in the presence of MSD cracks on the crack growth of the eccentric crack itself.

**Implementation**

The prediction of crack and delamination growth rates of an isolated crack in a FML is a simultaneous analysis of metal crack growth, interface delamination extension and their interaction. After each crack growth step, the growth rates need to be recalculated and the damage configuration must be updated. Therefore, the modelling of the fatigue damage evolution in FMLs is an iterative process [19].

For the analysis of crack growth in a FML panel with MSD cracks, the crack state of each crack has to be calculated. The effect of the presence of other adjacent cracks on each crack needs to be analysed by means of load redistribution mechanism at first, then the LEFM method together with principle of displacement compatibility is used to calculate the crack state. This process has to be repeated until all the crack states have been analyzed. Then the crack lengths and delamination shapes of all cracks are updated by using the parameters of each crack state. Figure 10 illustrates the overview of these two iterative processes and the essential parameters to be calculated.

The layup and the material properties of the FML to be investigated together with loading and temperature parameters are given as inputs. With these inputs, the stresses and corresponding strains in different layers in FMLs due to applied far-field load can be calculated by applying the Classical Laminate Theory (CLT) [2].

The MSD configuration in the FML can be initialized by specifying the number of cracks, $k$, locations of the two crack tips and two saw-cut tips for each crack with respect to a specified coordinate in the panel, and the size and shape of each delamination. A crack can be referred to as Crack $j$, where index $j$ ranges from 1 to $k$. In order to carry out the calculation of bridging stress distribution, the crack lengths and delamination shapes are discretized, specifying the locations where the displacement compatibility principle is implemented.

To aid in the calculation of the state of Crack $j$, the far applied load is decomposed into two parts to perform the calculation of the load redistribution mechanism and the bridging mechanism separately: the applied load carried by the intact bridging fibres over Crack $j$, and load in the remaining material. This decomposition follows the same principle in [9].

The load redistribution calculation is performed to evaluate the effect of other cracks on Crack $j$. The stress intensity factors $K_{red}$ for two crack tips of Crack $j$ due to the applied load and transferred load is determined. The load redistribution also affects the COD of Crack $j$, which is incorporated in bridging calculation. The bridging calculation is solving a system of equations representing the displacement compatibility principle at discretized nodes / bar elements for Crack $j$ only. With the bridging loads known, $K_{br}$ and $G$ for Crack $j$ can be determined. Then the crack growth and delamination extension rate is determined based upon the experimentally determined Paris relation. All these calculations need to be repeated for every crack until all the crack states are known.

After all the states of cracks in the FML panel have been analyzed, the length of each crack, $\Delta a$, and height of each delamination bar element, $\Delta b$, are updated based on the growth rates and applied cycles which is determined by the limited crack extension and the maximum crack growth rate. The increments of cracks and delamination shapes are updated iteratively.
The completion criteria can be link-up of two crack tips, the maximum crack length, maximum number of iterations, or maximum life cycles, $N$. If one of the completion criteria is met, the data can be output and post processed for analysing.

**PIN-LOADING EFFECT ON CRACK GROWTH BEHAVIOUR IN FMLs**

The mechanically fastened joints in fuselage structures are susceptible to MSD problem [6, 20, 21]. The dominant load for fuselage structures is introduced by the Ground-Air-Ground (GAG) pressurization cycles which create roughly constant amplitude fatigue loading. The load is transferred from one fuselage skin panel to another via the fasteners in the fastened joints. Normally the load path in fuselage joints is eccentric, which introduces secondary bending. The total

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**Figure 10. Prediction process of crack growth and delamination extension of MSD in FMLs**
stress in joints therefore comprises membrane stress, the secondary bending stress and the bearing stress resulted from pin-loading on the holes [22]. The stress concentrations caused by the presence of the fastener holes, pin-loading and secondary bending in fuselage joints affect the fatigue life of the joints [23]. Due to the fatigue loading and stress concentration contributors in fuselage joints, cracks can initiate at edges of multiple holes in the joints forming MSD problem.

The primary objective of investigating MSD is to combat MSD problem in fuselage joints to ensure the flight safety. Addressing MSD issue in fuselage joints without understanding the effect of pin loading on the crack growth behaviour is impossible. Since the proposed philosophy of MSD analysis for FML panels mainly utilizes the superposition principle, the effect of pin loading on the crack growth behaviour in FMLs can be superimposed onto the methodology for predicting MSD to tackle the MSD analysis in FMLs joints. The secondary bending effect can be incorporated by applying the methodology in [19]. A model for analysing pin loading effect on crack growth behaviour in FMLs therefore needs to be developed in order to build up an integral model for combating MSD in FML joints.

LIMITATION OF THE PROPOSED METHODOLOGY

In reality, the stress concentration is severe in front of the crack tips of MSD cracks. Plastic zones are also present in corresponding to the extremely high stress concentration. Neglecting the stress concentrations and plastic zones in front of adjacent cracks in idealization of crack interaction may lead to some error. The growth rate of a single crack before link up can be over predicted provided that the stress concentrations at adjacent crack tips are neglected. The over predicted growth rates lead to conservative results. When the crack tips get closer enough and the plastic zones start to interact, neglecting the influence of stress concentration and plastic zones will result in under prediction of crack growth rate. However, the error is acceptable according to the nature of fatigue in FMLs.

CONCLUSION

The nature of crack growth in FMLs results in different factors to be considered in MSD analysis. Larger possible damage sizes before link-up in FMLs result in the significance of load redistribution in predicting metal crack and interface delamination growth. The interaction between cracks caused by plasticity or stress concentration can be neglected without leading to much error in terms of total fatigue life.
Damage tolerance analysis for FMLs is based on simultaneous prediction of metal crack and interface delamination growth and the compatibility of metal crack and delamination. Extending this approach to simultaneous prediction of all crack states of multiple cracks and their compatibilities in FMLs is computationally inefficient and problematic. As a result, a new analysis methodology for FMLs containing MSD scenario is proposed in this paper based on idealization of the crack interaction in FMLs as load redistribution mechanism. The idealization is consistent with the nature of crack growth in FMLs.

This paper proposes the philosophy of MSD analysis for FMLs in order to develop a simplified methodology for predicting MSD growth in FML structures. Load redistribution mechanism for this simplified methodology has been developed. However, the analysis of eccentric crack growth in FMLs still needs to be developed. The model for eccentric crack in FMLs should be able to be extended to fit in with the MSD analysis. The methodology can be implemented according to Figure 10. In order to predict MSD growth in FML joint, the effect of pin loading effect on crack growth in FML panels needs to be studied. As the methodology is based on principle of superposition and LEFM, the investigation of pin loading effect can be integrated.

REFERENCE