Misinterpreting the Results: How Similitude can Improve our Understanding of Fatigue Delamination Growth

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Abstract: Use of the strain energy release rate for characterizing delamination growth in composite and bonded structures is now commonplace. Although its use is common, a consensus on the best formulation of strain energy release rate has not been reached for describing fatigue delamination growth. Most commonly selected formulations are the maximum strain energy release rate and the strain energy release rate range. This paper examines the use strain energy release rate range and how a misconception of its definition is misleading our understanding of fatigue delamination growth.

Keywords: Delamination, fatigue, strain energy release rate, similitude

1. INTRODUCTION

The study and characterization of delamination growth behaviour has gained significant attention in the research community due to the growing use of composite structures in a wide range of structural applications. A widely utilized approach is to apply a linear elastic fracture mechanics (LEFM) description using the strain energy release rate \( G \) in a manner analogous to metal crack growth in terms of the stress intensity factor \( K \). The primary reason for the use of \( G \) rather than \( K \) is that the local crack-tip stresses used to determine \( K \) are difficult to obtain in inhomogeneous composite laminates [1, 2]. Use of \( G \) avoids the need to determine crack-tip stresses.

Based on an analogy with the Paris relation for metal crack growth, delamination growth rate has been experimentally observed to have a linear relationship on a log-log scale when plotted vs. different formulations of the strain energy release rate, \( G \). What has resulted is a consensus that for fatigue delamination growth \( (db/dN) \):

\[
\frac{db}{dN} = C\left[f(G)\right]^n
\]

with \( C \) and exponent \( n \) being material constants. No consensus, however, has been reached on the formulation of \( f(G) \), resulting in it is simply being viewed as a curve fitting parameter that can be manipulated to best fit the experimental data. By far the most prevalent formulations used in the literature are \( f(G) = G_{\text{max}} \) [3, 4] based on the importance of this parameter for static delamination growth, and \( f(G) = \Delta G \) (where \( \Delta G = G_{\text{max}} - G_{\text{min}} \)) [1, 5-14] based on an analogy with the use of \( \Delta K \) for fatigue crack growth in metals. Both of these parameters can result in misleading results for fatigue delamination growth.

Examples of potentially misleading results can be illustrated with data from the literature. Figure 1 shows an example of Mode I fatigue delamination growth data for...
various R-ratios obtained by Hojo et al. [7] for a unidirectional carbon epoxy laminate plotted vs. $G_{\text{max}}$ and $\Delta G = G_{\text{max}} - G_{\text{min}}$. The variation in behaviour for Figure 1a is not unexpected as fatigue is a result of cyclic loading, and use of $G_{\text{max}}$ does not distinguish between the different minimum load cycles for the various R-ratios. The use of $\Delta G = G_{\text{max}} - G_{\text{min}}$ in Figure 1b attempts to remedy this by using the applied strain energy release rate range. A reduction in the variation between the various R-ratios is indeed observed, however, there is still a clear R-ratio dependency. Furthermore, this dependency seems counter-intuitive. For the same $\Delta G$, a higher delamination growth rate is observed for a lower R-ratio, and thus lower mean stress. In other words, a reduction in load results in increased fatigue damage!

Figure 1: R-ratio dependency observed in Mode I delamination growth in unidirectional carbon-epoxy laminates (reproduced using data from [7])

Figure 2 shows Mode I fatigue delamination growth results for a single Double Canteliver Beam (DCB) test of a unidirectional carbon-epoxy laminate carried out by Khan et al. [11]. Khan performed the fatigue test using a constant cyclic displacement at an R-ratio of 0.35. Once the delamination stopped growing (due to a decrease in $G$ with delamination length) Khan increased the opening displacement and resumed testing under the same R-ratio. The two tests performed on the same specimen under the same R-ratio resulted in two clearly distinct curves, suggesting that the delamination growth resistance for the single specimen changed between tests!
This paper will examine how the use of \( f(G) = G_{\text{max}} - G_{\text{min}} \) can lead to misleading interpretations of experimental data, similar to those illustrated in the above two examples. The source of these misinterpretations stems from the fact that the definition of \( \Delta G = G_{\text{max}} - G_{\text{min}} \) violates the underlying principle used to describe fatigue crack growth: the principle of similitude. A similitude based definition of \( \Delta G \) will be presented and how the use of this similitude based definition can improve our understanding of delamination growth will be discussed.

2. \textbf{Similitude as Applied to Metal Crack Growth}

The fundamental idea behind the principle of similitude is related to the fact that the size of an object is relative and physical phenomena within an object can be reduced to size independent relations [16, 17]. This principle is used regularly in engineering to describe physical phenomena. A simple example is the Hooke’s Law relation which relates force-deflection behaviour in a structure based upon the size independent similitude parameters of stress and strain. Using these similitude parameters, a size independent material constant, elastic modulus, can be defined.

The principle of similitude can also be applied to fatigue crack growth in which the crack growth behaviour is reduced to material behaviour independent of crack length. To do so, a similitude parameter must be defined which relates the conditions for similitude between the various crack geometries. The driving force for growth of a crack (either static or fatigue) is related to the stress state ahead of the crack tip; thus similitude implies that for a similar stress state ahead of a crack tip (regardless of crack length) similar crack growth behaviour should be observed. In metals, we represent this stress state by the linear elastic stress intensity factor, \( K \), which becomes the basis for similitude in crack growth analysis. It should be noted that the use of \( K \) is a simplification as the stress state in front of a crack is not linear elastic.

Defining \( K \) as a similitude parameter for crack growth, we can now differentiate between static and fatigue crack growth. Static crack propagation is related to a critical stress state in front of the crack tip and is thus propagation occurs when a limit or critical \( K \) value is reached. Fatigue is related to the cyclic stress state in front of the
crack tip, thus the similitude parameter for fatigue becomes $\Delta K = K_{\text{max}} - K_{\text{min}}$. Experimental observation has shown that for fatigue, the crack growth rate follows a linear trend, when plotted on a log-log scale vs. $\Delta K$, for the majority of the crack growth life (the so-called Paris region). This has led to the commonly applied Paris relation for describing fatigue crack growth:

$$\frac{da}{dN} = C_{cg} (\Delta K)^{n_{cg}}$$

where the material constants $C$ and $n$ have been given the subscript $cg$ to distinguish them from $C$ and $n$ for delamination growth. Numerous other formulations have been proposed to describe crack growth that includes the two asymptotes bounding the Paris region (defined by the threshold $\Delta K$ for delamination growth and the critical $K$ for static fracture), however these unnecessarily complicate the objective of this paper. Only the Paris region of growth will be discussed.

The reality for metals is that $C_{cg}$ and $n_{cg}$ are not simple material constants. The most notable case is the variation in crack growth behaviour (and thus $C_{cg}$ and $n_{cg}$) for fatigue loading under various R-ratios. This variation is not a result of a failure of the similitude principle, rather it arises due to the simplification in characterizing the crack growth driving force by a linear elastic description ($K$) of the stress state ahead of the crack tip. Knowing this has allowed mechanisms that alter the crack driving force which are not captured by $\Delta K$ to be identified. In the case of the R-ratio dependency in metals, that effect is related to variations in the amount of crack tip plasticity with varying R-ratio, and the reduction in crack growth driving force related to plasticity induced crack closure in the wake of the crack. Identifying this mechanism has permitted the continued use of a linear elastic stress intensity factor description of crack driving force with a correction related to the crack closure mechanism. Recognizing that for closure, there is no crack growth driving force when the crack tip is closed, similitude can be redefined based on the stress intensity factor range between maximum crack opening and crack closure:

$$\frac{da}{dN} = C (\Delta K_{\text{eff}})^n = C (K_{\text{max}} - K_{\text{close}})^n$$

where $K_{close}$ is the linear elastic stress intensity factor which is calculated for the tensile stress at which the crack tip closes.

This brief description of the characterization of metal crack growth is necessary as it has shaped our approach for characterizing fatigue delamination growth. Analogous to the use of $\Delta K = K_{\text{max}} - K_{\text{min}}$ for metal crack growth, the linear elastic parameter $\Delta G = G_{\text{max}} - G_{\text{min}}$ is often used to characterize fatigue delamination growth behaviour. Unlike for metal crack growth, descriptions of variations in delamination growth behaviour have remained mainly phenomenological, relying on empirical correction factors with little or no relation to physical mechanisms which influence similitude. It is the authors’ opinion that this lack of a mechanistic understanding of fatigue delamination growth is related to the use of an incorrect definition of $\Delta G$ which is unknowingly violating similitude.
3. **Similitude Based Definition of \( \Delta G \)**

The intent in using \( \Delta G \) in analogy to \( \Delta K \) as a fracture mechanics parameter for delamination growth is not in itself incorrect. Indeed, in using \( \Delta G \), researchers have been attempting to introduce the similitude principle into their characterization of fatigue delamination growth. They have been attempting to represent the cyclic state at the crack tip. The error in the application of \( \Delta G \) has been in its formulation as \((G_{\text{max}} - G_{\text{min}})\). In order to demonstrate this, we must examine what \( G \) is.

The strain energy release rate, \( G \), is equal to the total amount of elastic energy made available for crack (or delamination) extension per unit increase in crack surface area. Written as an energy balance:

\[
G = \frac{d}{db} (F - \Delta U) \tag{4}
\]

where \( F \) and \( \Delta U \) are the work applied to the system and increase in elastic energy of the system respectively, both as a result of an extension of the delamination by length \( b \).

It is common in a linear elastic fracture mechanics approach to subdivide \( G \) (or \( K \)) into components associated with three modes of crack surface deformation: Mode I (opening mode), Mode II (shear mode), and Mode III (tearing mode). Using linear elastic fracture mechanics it can be demonstrated that there is no interaction between these three modes. Thus, total \( G \) or total \( K \) can be obtained by superposition of the \( G \) or \( K \) components due to each mode \([18, 19]\).

\[
G = G_I + G_{II} + G_{III} \tag{5}
\]

It is this similarity between \( G \) and \( K \) that can lead to confusion. Although direct superposition of the different modes is possible, superposition within a single mode is different when considering \( G \) and \( K \). The principle of superposition pertains to the ability to subdivide a problem based on stresses and applied loads into smaller sub-problems that can be added together. This principle applies for a linear elastic system due to the linear relationship between stress and strain (and thus load and displacement). Because \( K \) by definition is a factor that is linearly related to the linear-elastic stresses, superposition of \( K \) implies:

\[
K = K_I + K_{II} + K_{III} \tag{6}
\]

\[
K_I = K_{I(1)} + K_{I(2)} + K_{I(3)} + \cdots
\]

\[
K_{II} = K_{II(1)} + K_{II(2)} + K_{II(3)} + \cdots
\]

\[
K_{III} = K_{III(1)} + K_{III(2)} + K_{III(3)} + \cdots
\]

Strain energy release rate, \( G \), however, is not linearly related to the linear-elastic stresses. This can also be illustrated by examining the relationship between \( K \) and \( G \). This relationship can be explicitly defined for the case of a Mode I crack in an isotropic material under plane strain or plane stress as:
\[ G_f = \frac{K_i^2}{E'} \]  

(7)

where \( E' = E \) for plane stress and \( E/(1 - \nu) \) for plane strain. In general, however, \( G \propto K^2 \) for other loading states and for isotropic and anisotropic materials [18-20]. Combining this proportionality with the results of equation (6), we obtain the following expression for superposition of \( G \) [19]:

\[ G = G_I + G_{II} + G_{III} \]

\[ G_I = \left[ \sqrt{G_{I(1)}} + \sqrt{G_{I(2)}} + \sqrt{G_{I(3)}} + \cdots \right]^2 \]

\[ G_{II} = \left[ \sqrt{G_{II(1)}} + \sqrt{G_{II(2)}} + \sqrt{G_{II(3)}} + \cdots \right]^2 \]

\[ G_{III} = \left[ \sqrt{G_{III(1)}} + \sqrt{G_{III(2)}} + \sqrt{G_{III(3)}} + \cdots \right]^2 \]

(8)

This result suggests that the proper formulation of \( \Delta G \) which obeys the rules of superposition is given as follows:

\[ \Delta G = \Delta G_I + \Delta G_{II} + \Delta G_{III} \]

\[ = \left( \sqrt{G_{I_{\text{max}}} - \sqrt{G_{I_{\text{min}}}}} \right)^2 + \left( \sqrt{G_{II_{\text{max}}} - \sqrt{G_{II_{\text{min}}}}} \right)^2 + \left( \sqrt{G_{III_{\text{max}}} - \sqrt{G_{III_{\text{min}}}}} \right)^2 \]  

(9)

A simple way to illustrate the impact of using \( \Delta G = G_{\text{max}} - G_{\text{min}} \) vs. the definition in equation (9) is to consider a case where delamination behaviour for two load ratios are obtained for the same crack/specimen geometry. For this illustration, we will consider a double cantilever beam (DCB) specimen with a delamination crack length, \( b \), and specimen width, \( B \). For such a specimen, the Mode I strain energy release rate can be determined by [18]:

\[ G = \frac{P^2}{2B} \frac{dC}{db} \]  

(10)

where \( P \) is the applied load, \( B \) is the specimen width, and \( C \) is the compliance of the DCB specimen. If the crack and specimen geometries are identical, then the specimen compliance is the same, the crack tip stress field (characterized by \( K \)) is only a function of applied load, and differences in \( G_I \) are influenced by the square of the applied load only. By constraining the comparison to the same specimen dimensions and same crack geometry, we effectively reduce similitude to the applied load cycle.

If we consider this simple DCB example and determine what applied load cycles would produce the same \( \Delta G \) for different R-ratios (considering crack and specimen geometry is kept constant), we can easily see the loss in similitude when using \( \Delta G = G_{\text{max}} - G_{\text{min}} \). Since for \( R = 0 \), \( \Delta G = G_{\text{max}} \) regardless of which formulation of \( \Delta G \) is used, it will serve as a datum for comparison. The load cycle to produce an equivalent \( \Delta G \) based on the two different formulations is shown in Figure 3. It is clear that the load cycle amplitude changes for \( R = 0.3 \) when \( \Delta G = G_{\text{max}} - G_{\text{min}} \), which is a violation of the similitude principle.
Figure 3: Schematic of the applied stress amplitudes for a constant $dC/da$ and $\Delta G$ using arithmetic and superposition definitions of $\Delta G$.

This illustration restricted itself to comparing the same crack geometry at different $R$-ratios, but the benefit of using a similitude based definition of $\Delta G$ can be further extended. Consider the second example given in the Introduction where DCB test data produced by Khan resulted in two different delamination growth resistances within the same specimen. In this instance, the $R$-ratio remained constant, but once growth of the delamination arrested under the initial loading conditions, the applied load was increased (by means of the opening displacement). This permitted measurements of delamination growth to be made for larger delamination lengths. When this data is plotted using $\Delta G = G_{\text{max}} - G_{\text{min}}$, an apparent change in the delamination growth resistance is observed. If this data is replotted using the similitude definition of $G$, this difference is significantly reduced to a level that could be explained by scatter in the measurements (see Figure 4).
Figure 4: Effect of application of similitude based definition of $\Delta G$ on Mode I delamination growth data of Khan (based on data from [11])

4. CONSEQUENCES OF USING A NON-SIMILITUDE DEFINITION OF $\Delta G$

This section will attempt to examine the implications of using the incorrect formulation of the strain energy release rate range on interpreting fatigue delamination growth using data from the literature. Although this will be done in a general manner, the authors caution the reader that trends in delamination growth can be highly material specific. The point of this section is not to make claims that all materials behave one way with respect specific issues discussed, such as R-ratio dependency and mixed mode growth. The objective of this section is to demonstrate that using a sound definition of the strain energy release rate range can result in a more meaningful and useful analysis of fatigue delamination growth data.

4.1 R-Ratio Effects

Experimental evidence has demonstrated that the ratio of minimum and maximum applied stress (R-ratio) has an influence on both crack growth in metals and delamination growth in composites when plotting vs. the stress intensity factor range, $\Delta K$ [7, 21]. Strictly speaking, the principle of similitude implies that such an effect should not be present. The same amount of growth should occur for the same crack tip conditions (characterised by $\Delta K$ in this case). This seeming contradiction can be explained by the fact that although $\Delta K$ is a similitude parameter, it is a calculated parameter that carries along certain assumptions with it. In this case, $K$ is a linear elastic parameter, so any deviation from a linear elastic stress state can introduce a misrepresentation of the similitude of the behaviour. This is precisely what happens for crack closure behaviour in metals. Plasticity in the wake of the crack and/or fracture surface roughness can cause closure of the crack tip at non-zero loads, resulting in a change in the crack tip stress state from that characterized by the linear elastic fracture mechanics parameter, $K$.

Use of a linear elastic fracture mechanics similitude parameter, however, is still useful. In cases where violations in similitude based on linear elastic assumptions are observed, potential mechanisms which violate those assumptions can be identified. Furthermore, the impact of that mechanism can be quantified based on the deviation from similitude defined using a linear elastic fracture mechanics parameter. This is
illustrated by the crack closure example in metals where the observed R-ratio effect for Mode I crack growth can be used to estimate the magnitude of $K_{\text{close}}$ required to account for the effect. This value can then be investigated through dedicated crack closure tests [22].

The same benefit for investigating fatigue delamination growth in composites can be realized when using a sound definition of $\Delta G$, based upon similitude and the rules of superposition for $G$. To illustrate this, the fatigue delamination growth data obtained by Hojo et al. [7] that was introduced in Figure 1, will be used. Hojo characterized the delamination growth behaviour of laminates made with Toray P305 prepreg (T300 carbon fibres combined with #2500 epoxy resin) under Mode I loading using double cantilever beam (DCB) specimens at R-ratios of 0.2, 0.5, and 0.7. In addition to characterizing the growth results in terms of $\Delta G = G_{\text{Imax}} - G_{\text{Imin}}$, Hojo also plotted the results in terms of $\Delta K_i$, both of which are shown in Figure 5.

![Figure 5: Comparison of R-ratio effect on Mode I fatigue delamination growth in unidirectional carbon epoxy laminates using $\Delta K_i$ and $G_{\text{Imax}} - G_{\text{Imin}}$ as characterization parameters (based on data from [7])](image)

An immediate difference is seen in Figure 5 for the fatigue delamination growth behaviour using the two characterization parameters. With $\Delta K$ representing the driving force for delamination growth, it is observed that for the same delamination growth rate at different R-ratios, a lower $\Delta K$ is required at higher R-ratios. This suggests some sort of mean stress or peak stress dependency on the growth rate. With $\Delta G = G_{\text{Imax}} - G_{\text{Imin}}$ representing the driving force for delamination growth, the opposite trend is observed. This should be counter intuitive for two reasons. First, $\Delta K$ and $\Delta G$ are related parameters, where the use of $\Delta G$ is more prevalent due to ease of calculation for anisotropic composite materials, and as a result of their relation to each other, should exhibit similar (and certainly not opposite) trends. Second, the results in Figure 5b indicate that an increase in R-ratio (realized through an increase in mean applied stress) results in a necessary increase in $\Delta G$ in order to realize the same crack growth behaviour as lower R-ratios. In other words, delamination growth resistance increases with increasing mean stress.

If we replot Hojo’s data using $\Delta G$ based upon the rules of superposition of $G$, Figure 6 is obtained. Comparison of Figure 6 with Figure 5a shows a consistent trend in the R-ratio effect when using this formulation of $\Delta G$. Although there is greater scatter in
the data between the different R-ratios than in Figure 5b, the comparison of the R-ratios is done based on linear elastic similitude, and thus the causes for the differences between the R-ratios can be investigated based on this fact.

Figure 6: Replot of Mode I carbon/epoxy fatigue delamination growth data from Hojo [7] using \( \Delta G = (\sqrt{G_{\text{I,max}}} - \sqrt{G_{\text{I,min}}})^2 \) (based on data from [7])

It may appear from simple comparison of the two plots in Figure 5 that, despite the violation of similitude when using \( \Delta G = G_{\text{I,max}} - G_{\text{I,min}} \), there is a significant reduction in scatter between the different R-ratios when using this formulation of \( \Delta G \). Indeed this is apparent, but this is a result of the formulation of \( \Delta G \) inadvertently including an R-ratio dependency. This can be observed by expanding this formulation using equation (9):

\[
G_{\text{I,max}} - G_{\text{I,min}} = \Delta G^* + 2R\sqrt{\Delta G^*}\sqrt{G_{\text{I,max}}}
\] (11)

where \( \Delta G^* \) is the formulation of \( \Delta G \) based on superposition as given in equation (9). The inclusion of the R-ratio dependency seems to reduce the scatter of the data, but it is not based on any understanding of the R-ratio effect, and thus is purely coincidental. A more rigorous approach to the study of delamination growth phenomenon using a similitude based parameter may result in a better understanding of the R-ratio effect, and could result in an understanding based correction, as was the case for crack closure in metals.

Unfortunately, the opposite is occurring. In a recent paper by Jia and Davalos [8], the authors attempted to investigate different formulations of strain energy release rate used in a Paris relation for characterizing the R-ratio effects of Mode I delamination growth. The different formulations were based on a direct analogy to different formulations of stress intensity factor studied for crack growth in metals such that:

\[
\frac{da}{dN} = C(f(K))^n \rightarrow \frac{da}{dN} = C(f(G))^n
\]

Table 1 summarizes the formulations attempted by Jia and Davalos, including the stress intensity factor formulation used as an analogy and the appropriate definition of \( f(G) \) based on the superposition rules of \( G \) and the analogy with \( f(K) \).
Table 1: Comparison of Jia and Davalous formulations for G with K based analogy and equivalent superposition based formulations for G

<table>
<thead>
<tr>
<th>f(G) As formulated by Jia and Davalous</th>
<th>f(K) Used as an analogy</th>
<th>f(G) Based on K analogy and superposition rules for G</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta G = G_{\text{max}} - G_{\text{min}} )</td>
<td>( \Delta K )</td>
<td>( \left( \sqrt{G_{\text{max}}} - \sqrt{G_{\text{min}}} \right)^2 )</td>
</tr>
<tr>
<td>( G_{\text{max}}^2 - G_{\text{min}}^2 = 2 \left( G_{\text{max}} - G_{\text{min}} \right) \left( \frac{G_{\text{max}} + G_{\text{min}}}{2} \right) )</td>
<td>( K_{\text{max}}^2 - K_{\text{min}}^2 = 2 \Delta KK_{\text{mean}} )</td>
<td>( G_{\text{max}} - G_{\text{min}} )</td>
</tr>
<tr>
<td>( G_{\text{avg}} = \left( \frac{G_{\text{max}} + G_{\text{min}}}{2} \right) )</td>
<td>( K_{\text{avg}} )</td>
<td>( \left( \frac{\sqrt{G_{\text{max}}} + \sqrt{G_{\text{min}}}}{2} \right)^2 )</td>
</tr>
</tbody>
</table>

The results in Table 1 illustrate the potential for misunderstanding when incorrectly dealing with the addition and subtraction of G. In attempting to make an analogy to \( \Delta K \) with their formulation of \( \Delta G = G_{\text{max}} - G_{\text{min}} \), Jia and Davalous were actually making an analogy with \( K_{\text{max}}^2 - K_{\text{min}}^2 \). Such errors are common in the literature and may be hampering our understanding of delamination growth.

The above point can be illustrated using another case study from the literature. Kawashita et al. [9] performed Mode II static and fatigue delamination tests on unidirectional cut-ply specimens made of IM7/8552 carbon/epoxy prepreg. Results from these tests as plotted by Kawashita et al. are given in Figure 7a, illustrating the substantial R-ratio dependency they reported. This data has been replotted in Figure 7b using a similitude based definition of \( \Delta G \), which results in all of the data collapsing to a single trend line. The exception being two data points for \( R = 0.5 \); however, a quick calculation reveals that \( G_{\text{max}} \approx G_C \) for these two points, indicating that the observed delamination growth could be influenced by both fatigue and static propagation.
The results in Figure 7 are not unique. The authors have examined many data sets for Mode II delamination growth in composites and hybrid structures [6, 13, 14, 23], and in all cases use of a similitude based definition of \( \Delta G \) resulted in a significant reduction, if not complete elimination, of the observed R-ratio effects. It seems that using a linear elastic based similitude parameter for characterizing Mode II delamination growth gives consistent results. Of course this will always be material dependent, however in cases where it is different, physical mechanisms responsible for the difference should be identified.

4.2 Residual Stresses

Residual stresses/strains form in composite laminates (and other bonded structures) during the cooling phase after curing due to mismatches in the thermal coefficients of expansion amongst the layers of the laminate. Several studies have investigated the effects of residual stress on delamination growth behaviour and have developed numerous parameters and methods for accounting for their effects [12-14, 24, 25]. All of these studies, however, have been influenced by the incorrect definition of \( \Delta G \).

If we consider the effect of residual stresses on the applied fatigue stress cycle, it is evident that the residual stress alters the mean fatigue stress, but not the stress amplitude, because the residual stress is present for the entire fatigue cycle. The presence of the residual stress effectively results in a change in R-ratio. Thus, the same points described in the previous section apply here for residual stresses as well. The use of \( \Delta G = G_{\text{max}} - G_{\text{min}} \) can result in misinterpretations of delamination growth data due to the violation of similitude.

To illustrate one such example, results from a study conducted by Lin and Kao will be used [13]. Lin and Kao investigated the effect of residual curing stresses on the Mode II delamination growth behaviour of the hybrid Fibre Metal Laminate (FML) concept. The particular variant studied consisted of alternating carbon/epoxy and aluminum layers. One unique property of the FML concept is that the material can be post-stretched after curing. Post-stretching introduces plastic strains in the metal layers that can be used to reduce, eliminate, or reverse the residual stress state depending on the amount of plastic strain introduced. Lin and Kao investigated the effects of residual stress by testing 3 laminate configurations, consisting of 2, 4, and 6 carbon-epoxy
prepreg sheets (designated C2, C4, and C6 respectively) between each pair of aluminum sheets, resulting in three different residual curing stress states. Furthermore, laminates were tested in the as-cured state containing the residual thermal strains, and in a post stretched condition where residual stresses had been removed. Tests were performed at an R-ratio of 0.1.

The data obtained by Lin and Kao is plotted in Figure 8a. It must be noted that in their paper, Lin and Kao neglected to account for the work done by the applied load (term $F$ in equation (4)) due to the release of residual strain that occurs during delamination propagation. This error, and the necessary correction, is detailed in reference [15]. Looking at the results in Figure 8a, it appears that there is a significant influence of residual stress on the delamination growth rate. All the results for post stretched laminates show consistent behaviour (partially a result that as the R-ratio approaches 0, $G_{\text{max}} - G_{\text{min}} \rightarrow (\sqrt{G_{\text{max}}} - \sqrt{G_{\text{min}}} )^2$). For the laminates containing residual stresses, a different behaviour is observed, and the behaviour of the 3 laminate configurations differ from each other (each has their own effective R-ratio). Using the similitude based definition of $\Delta G$ as done in Figure 8b, the data collapses to a single trend.

\begin{equation}
\Delta G = (\sqrt{G_{\text{max}}} - \sqrt{G_{\text{min}}} )^2
\end{equation}

\begin{equation}
\Delta G = \left[ \left( \sqrt{G_{\text{applied, max}}} + \sqrt{G_{r}} \right) - \left( \sqrt{G_{\text{applied, min}}} + \sqrt{G_{r}} \right) \right]^2
\end{equation}

where the removal of $G_{r}$ is only possible due to the rules of superposition for $G$ as given in equation (8).

Figure 8: Comparison of residual stress effects on Mode II delamination growth behaviour using $\Delta G = G_{\text{II,max}} - G_{\text{II,min}}$ and similitude based definition of $\Delta G$ (based on data obtained from [12, 13, 24, 25]).

Conveniently, the same results as shown in Figure 8b can be obtained when eliminating the inclusion of residual stresses in the calculation of $G$. If the total strain energy release rate, $G$, is broken up into a component due to the residual stress ($G_{r}$) and a component due to the applied loads ($G_{\text{applied}}$), then:

\begin{align*}
\Delta G &= \left( \sqrt{G_{\text{max}}} - \sqrt{G_{\text{min}}} \right)^2 \\
&= \left[ \left( \sqrt{G_{\text{applied, max}}} + \sqrt{G_{r}} \right) - \left( \sqrt{G_{\text{applied, min}}} + \sqrt{G_{r}} \right) \right]^2 \\
&= \left( \sqrt{G_{\text{applied, max}}} - \sqrt{G_{\text{applied, min}}} \right)^2
\end{align*}
This removal of the residual stress component of the strain energy release rate is convenient as it is often difficult to accurately determine the residual stress state due to the limited accuracy of the coefficients of thermal expansion used to determine it. Thus, for fatigue delamination propagation purposes, they can be omitted from the analysis. In making such an omission, it should be remembered that the inclusion of the residual stress component of $G$ can not be omitted when considering static failure (residual stress affect $G_{\text{max}}$ and $G_{\text{min}}$, just not the similitude definition of $\Delta G$). Additionally, in materials/loading modes where the stress ratio has an effect on fatigue delamination growth behaviour, the change in stress ratio resulting from the residual stress state should be taken into account.

### 4.3 Mixed Mode Behaviour

Numerous studies into the mixed mode delamination growth behaviour of composites and bonded laminates have been performed in the literature [1, 10, 26, 27]. A common approach taken in these studies is to characterize the delamination growth behaviour in terms of $\Delta G = G_{\text{max}} - G_{\text{min}}$ for various different mode mixities, obtain Paris relation constants ($C$ and $n$ in equation (1)) for each of these data sets. These resultant Paris relation constants are then treated as function of mode mixity, and a general expression for each is determined by curve fitting the results from the various tests. This approach, although practical, does not provide much insight into the interaction (if any) of the various loading modes. Furthermore, the lack of similitude in applying $\Delta G = G_{\text{max}} - G_{\text{min}}$ also limits the understanding that can be derived from comparing results between the mode mixities as it creates a false basis for comparison.

A more rigorous approach to investigating mixed mode delamination growth would be to attempt to define the interaction (if any) between the different loading modes such that delamination growth data for the pure loading modes could be used to predict growth at an arbitrary level of mode mixity. This nature of this approach inherently requires the use of superposition and similitude, thus a definition of $\Delta G$ that complies with this is required.

To illustrate this approach, delamination growth data obtained by Kenane [10] for unidirectional glass/epoxy laminates will be used. Kenane obtained delamination growth data for pure Mode I and pure Mode II loading using DCB and ELS specimens respectively. Additionally, mixed-mode (I+II) data was also obtained using mixed-mode bending (MMB) specimens for several different ratios of mode mixity. All tests were performed at an R-ratio of 0.1. The resultant test data is given in Figure 9. For brevity, the data is only plotted in terms of the similitude based definition of $\Delta G$ as given in equation (9). To see the data plotted in terms of $\Delta G = G_{\text{max}} - G_{\text{min}}$, the reader is referred to reference [10].
For the pure Mode I and II data sets, the resultant Paris relation constants and threshold values calculated for the data sets are given in Figure 9a. It should be noted that these values are different from those given in Kenane’s paper due to the change in definition of $\Delta G$ used for plotting the results. Using these two sets of Paris relation constants, predictions of the mixed mode behaviour were made simply by dividing the strain energy release rate range into its Mode I and Mode II components:

$$\frac{da}{dN} = C_1 \left( \sqrt{G_{I_{\text{max}}} - G_{I_{\text{min}}}} \right)^{n_1} + C_2 \left( \sqrt{G_{II_{\text{max}}} - G_{II_{\text{min}}}} \right)^{n_2}$$  \hspace{1cm} (13)

where $C_1$ and $n_1$ are the Mode I Paris constants and $C_2$ and $n_2$ are the Mode II Paris constants. In instances where the individual component is below the observed threshold for that loading mode, the contribution for that mode is set to zero. These predictions are plotted alongside the experimental data obtained by Kenane for $G_{II}/G_T = 53\%$ and $82\%$. Excellent agreement between the predictions and mixed-mode data is obtained for this simple superposition approach, indicating that for this particular example that no interaction effects are observed between the two loading modes. This result provides better insight into the mixed-mode delamination growth behaviour than the curve fitting approach to determining mixed-mode Paris relation constants described earlier. Furthermore, it limits the required experimental data needed for prediction, even though the additional test data was valuable for verifying the assumption of no interactions between loading modes.

5. CONCLUSIONS
This paper has highlighted the error in defining $\Delta G = G_{\text{max}} - G_{\text{min}}$ due to a violation of the rules of superposition for $G$. The correct formulation for $\Delta G$ (given in equation (9)) provides a definition consistent with the rules of superposition and which maintains similitude when comparing delamination growth behaviour between different test/crack geometries.
Evidence was provided to demonstrate the potential pitfalls when using the incorrect formulation of $\Delta G$ when investigating fatigue delamination growth. A lack of similitude within that parameter can result in trends being observed that may be mistakenly attributed to test variables such as R-ratio and residual stress, which may actually be a result of the inconsistent comparison generated by the non-similitude parameter.

Finally, it may be argued that in adopting a Paris relation to fit experimental delamination growth data, the selection of the independent variable for fitting the data is arbitrary. It is merely an exercise in correlating experimental data. This, however, should not be the case. Applying a Paris relation is done not simply to curve fit experimental data, but to attempt to capture the material behaviour based on an assumed theory and the principle of superposition. In the case of delamination growth, that theory is linear elastic fracture mechanics. Thus, the selection of the independent variable should be based on this theory, and should conserve similitude and obey the rules of superposition outlined within it. Otherwise, efforts to characterize material behaviour are reduced to arbitrary curve fits with limited applicability and transferability.

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**References**


