On the Onset of the Asymptotic Stable Fracture Region in the Mode II Fatigue Delamination Growth Behaviour of Composites

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ABSTRACT: An experimental investigation aimed at identifying the presence and onset of an asymptotic stable fracture region in the mode II fatigue delamination growth behaviour of composites is presented. The study is motivated by the possibility that experimental data sets, particularly those obtained at high R-ratios, may unknowingly contain data points within the asymptotic stable fracture region that influence its perceived log-linear behaviour and the fit of various log-linear delamination growth models. Results from the experimental investigation indicate that the asymptotic stable fracture region can extend to $G_{\text{II临界}}$ values as low as 0.7$G_{\text{II临界}}$. The implications of this result on characterizing and fitting of various delamination growth models (including models assuming log-linear behaviour) to delamination growth behaviour is discussed.

KEYWORDS: Mode II, fatigue, delamination growth, asymptotic behaviour

NOMENCLATURE:

Symbols

\begin{itemize}
\item \textit{a} \hspace{1cm} \text{Crack or delamination length (mm)}
\item \textit{B} \hspace{1cm} \text{Specimen width (mm)}
\item \textit{E} \hspace{1cm} \text{Young’s modulus (GPa)}
\item \textit{G} \hspace{1cm} \text{Strain energy release rate (kJ/m}^2\text{)}
\item \Delta \textit{G} \hspace{1cm} \text{Strain energy release rate range (kJ/m}^2\text{)}
\item \text{(}\Delta \sqrt{\text{G}}\text{)}^2 \hspace{1cm} \text{Alternative definition of strain energy release rate range (kJ/m}^2\text{)}
\item \textit{K} \hspace{1cm} \text{Stress intensity factor (MPa}\sqrt{\text{mm}}\text{)}
\item \Delta \textit{K} \hspace{1cm} \text{Stress intensity factor range (MPa}\sqrt{\text{mm}}\text{)}
\item \textit{L}_{\text{gauge}} \hspace{1cm} \text{Extensometer gauge length (mm)}
\item \textit{N} \hspace{1cm} \text{Number of fatigue cycles}
\item \textit{P} \hspace{1cm} \text{Applied load (N)}
\item \textit{R} \hspace{1cm} \text{Fatigue stress ratio}
\item \textit{t} \hspace{1cm} \text{Specimen thickness (mm)}
\item \eta \hspace{1cm} \text{Relative error (\%)}
\item \textit{\chi} \hspace{1cm} \text{Ratio of cut plies to total plies}
\item \epsilon^* \hspace{1cm} \text{Extensometer strain (mm/mm)}
\end{itemize}

Subscripts

\begin{itemize}
\item \textit{1} \hspace{1cm} \text{Fibre direction}
\item \textit{2} \hspace{1cm} \text{Transverse fibre direction}
\end{itemize}
II Mode II
c Critical (in relation to static failure)
max Maximum (in relation to fatigue cycle)
min Minimum (in relation to fatigue cycle)

1 INTRODUCTION

The damage tolerance certification requirement for civil aircraft necessitates the ability to detect and repair structural damages before they become critical to the integrity of the aircraft. Two approaches to this requirement are typically employed. In the slow-growth approach, a slow and predictable fatigue growth life for possible damage scenarios must be demonstrated. Inspection intervals are set to ensure sufficient opportunity to detect and repair potential damages over their detectable growth life. In the no-growth approach, potential incidental damages (i.e. delaminations due to impact) must have a demonstrable lack of growth under service fatigue loads. Inspection intervals are based on the probabilities of specific damage causing incidents occurring over the lifetime of the structure. Regardless of the approach used, sufficient knowledge of the damage propagation behaviour under static and fatigue loading is required.

Delaminations are a particular damage type of concern for composite structures. Manufacturing defects and in service impact events have the potential to cause delaminations that significantly reduce the strength and stiffness of composite structure. Understanding the static and fatigue propagation behaviour of delaminations is thus critical for the application of composite structures in aircraft. A recent critical review of the literature presented by Pascoe et al.\(^1\) demonstrates the wide variety of modeling approaches that have been employed to help further that understanding on fatigue propagation behaviour.

It is generally accepted that fatigue delamination growth, analogous to metal fatigue crack growth, exhibits a sigmoidal behaviour when plotted against strain energy release rate on a log-log scale as illustrated in Figure 1. Numerous models for predicting growth focus on the log-linear region of the curve, also commonly known as the Paris region based on the early work of Paris et al.\(^2\),\(^3\). These models take a general form of the power law:

\[
\frac{da}{dN} = C \left[ f(G) \right]^n
\]  

(1)

Where \(da/dN\) is the delamination growth rate, \(C\) and \(n\) are empirical constants, assumed to be material constants, and \(f(G)\) is a formulation of the strain energy release rate. Common formulations include \(G_{\text{max}}\) and \(\Delta G\) as discussed in\(^1\).
The asymptotes for the threshold and stable fracture regions of Figure 1 are generally accepted to be defined by a threshold strain energy release rate or range ($G_{th}$ or $\Delta G_{th}$) and the critical strain energy release rate for static failure ($G_c$) respectively. Several modifications to the Paris relation in equation (1) have been proposed in the literature to capture the asymptotic behaviour in these regions. Martin and Murri 4 proposed multiplying the Paris relation by a sigmoidal factor dependant on $G_{th}$ and $G_c$. A similar approach was also adopted by Kinloch and Osiyemi 5, and Abdel Wahab et al. 6. Alternatively, modifying $f(G)$ in the Paris relation to introduce the sigmoidal behaviour has been adopted in models proposed by Jones et al. 7, Andersons et al. 8 (for threshold behaviour only), and in metal crack growth models proposed by Forman et al. 9. Both approaches rely on assuming a sigmoidal behaviour a priori and including factors to enforce this behaviour that are often “viewed as parameters that are used to ensure that the entire range of data fits the equations” 7 rather than physics-based quantities. Furthermore, these factors, dependent on the implementation, can change the underlying meaning of the material dependent constants $C$ and $n$.

Identification of the onset of each asymptotic region is important for the characterization and understanding of fatigue delamination behaviour. A relevant case study to illustrate this point is the study of R-ratio effects on fatigue delamination growth behaviour. In order to quantify the influence of R-ratio, fatigue delamination growth curves similar to that illustrated in Figure 1 are experimentally determined for constant R-ratios. Several curves are generated for different R-ratios and compared. When testing higher R-ratios, higher maximum fatigue loads are required to achieve comparable fatigue amplitudes as tested for lower R-ratios. In other words, $G_{max}$ is necessarily increased to achieve a comparable $\Delta G$. Thus, the threshold asymptote dependent on $\Delta G$ and the static failure asymptote dependent on $G_{max}$ will move closer to each other, narrowing the log-linear region of Figure 1, as the R-ratio is increased. As this narrowing effect increases, there is a risk that data points obtained from experiments that appear to maintain the log-linear trend when plotted actually lie within the asymptotic regions of the delamination growth curve.

The possibility of the above case occurring has been identified by the authors in three data sets from the literature. A recent paper by Rans et al. 10 discusses the formulation of the cyclic strain energy release rate range for delamination growth characterization and advocates the use of $(\Delta \sqrt{G})^2$ as strain energy release rate range consistent with similitude of the cyclic crack tip stress range. Several case studies presented by Rans et al. 10 demonstrated that use of this definition eliminated the apparent R-ratio effects observed when using $\Delta G$ or $G_{max}$ as the independent variable for characterizing Mode II fatigue delamination growth behaviour. The present authors have applied this definition of the strain energy release rate range to analyzing three new data sets from the literature 11-13. The results of these analyses are given in Figures 2 through 4, with subfigures a) depicting the data as a function of $G_{limax}$, and subfigures b) depicting the
data as a function of \((\Delta \sqrt{G_{II}})^2\). Results in subfigures a) illustrate the higher magnitudes of \(G_{\text{II max}}\) tested at higher R-ratios and how the results from each R-ratio appear to be log-linear. Plots of the data reanalyzed using \((\Delta \sqrt{G_{II}})^2\) in subfigures b) indicate another possibility. The data sets correlate well with each other using this parameter except for a few data points indicated with label A. These data points all correspond to tests run at higher R-ratios and at the highest \(G_{\text{II max}}\) values examined in their respective studies, and for all the studies fall within the range \(G_{\text{II max}} > 0.7G_{\text{II c}}\). It is possible that these data points lie within the asymptotic stable fracture region, but that this is masked by a combination of a lack of additional data points in this region, scatter in the data, and the ability for log-log scales to mask subtle deviations in trends.

The results in Figures 2 through 4 motivate the need to study the onset of the stable fracture region for fatigue delamination growth. This paper presents an experimental study on mode II delamination growth in a unidirectional carbon/epoxy material in response to this need.
Figure 4: Experimental fatigue delamination growth data for E-glass/913; label A indicates data points where $G_{II_{\text{max}}} > 0.7G_{IIc}$

2 TEST PROGRAM

Based upon the above evidence from the literature, a test program to investigate the onset of the asymptotic stable fracture region in the mode II fatigue delamination growth behaviour for composite materials was carried out. This test program consisted of mode II fatigue delamination growth tests performed on a unidirectional carbon/epoxy material under constant amplitude loading at various R-ratios and applied loads. This section summarizes the test program, including specimen configuration, measurement methods, and error analysis.

2.1 The Central Cut-Ply Specimen

Fatigue delamination growth tests were carried out with the so-called central cut-ply specimen used in previous studies of Mode II fatigue delamination growth in composite materials. A similar specimen configuration, with a discontinuous metal layer, has also been used extensively to study Mode II fatigue delamination growth in hybrid Fibre Metal Laminate materials. The test specimen consists of a series of discontinuous (or cut) plies laminated between continuous plies in a symmetric layup.

Four delamination fronts initiate from the discontinuity in the laminate producing planar delaminations along the two interfaces between the continuous and discontinuous plies (Figure 5). A pure mode II shear field is applied to the delamination fronts through a far field tensile force, $P$, applied to the ends of the specimen. An expression for the strain energy release rate associated with each of the four delamination fronts was determined by Allegri et al., following the approach outlined by Williams, and is summarized below.

$$G_{II} = \frac{P^2}{4B^2Et} \left( \frac{\chi}{1-\chi} \right)$$ (2)

where $\chi$ denotes the ratio of the number of cut plies to the total number of plies in the specimen, $B$ and $t$ are the specimen width and thickness respectively, and $E$ is the Young’s modulus of the material.
Equation (2) holds only if the delamination state is symmetric across the specimen mid-plane, and if all four delaminations grow at the same rate. The delamination state does not need to be symmetric with respect to the cut-ply location, as illustrated in Figure 5, because the strain energy release rate is independent of delamination length.

The independence of equation (2) on delamination length is a desirable property for fatigue delamination growth characterization. It permits delamination growth measurements at a constant strain energy release rate through simple constant amplitude fatigue tests. This strain energy release rate solution, however, is based upon simple beam theory and the assumption that the delaminated plies are fully unloaded. These assumptions break down as the delamination length approaches zero, resulting in a region where equation (2) is not valid. FE simulations performed by Kawashita et al. have shown that this region encompasses delamination lengths up to a length of five to ten times the thickness of the cut plies. All tests carried out in the present test program do not consider delamination growth within this region.

2.2 Delamination Growth Measurement Approach

Monitoring of delamination length during the fatigue test can be achieved by monitoring changes in specimen compliance using an extensometer. As the delaminations in the specimen grow, the overall specimen stiffness reduces. This stiffness reduction can be modelled by dividing the specimen into a series of beams. Pristine regions of the specimen will have a stiffness equal to the nominal specimen stiffness, while delaminated regions will have their stiffness reduced by a factor of $(1 - \chi)$. The total deformation of the beam model is thus a function of delamination length, applied load, and the nominal specimen stiffness. Equating the deformation from the model to the deformation measured by the extensometer, the size of the delamination present in the specimen can be estimated. For brevity, the final expression for delamination length is provided below.

\[
gauge L 2a = L_{gauge} \frac{1 - \chi}{\chi} \left( \frac{EBt}{P} \varepsilon^* - 1 \right)
\]

where $L_{gauge}$ and $\varepsilon^*$ are the extensometer gauge length and strain measurement respectively. A detailed account of the derivation was presented by Allegri et al. This equation is valid for the scenario where both planar delaminations of length $2a$ are symmetric across the laminate mid-plane.

Direct application of equation (3) to determine delamination length is highly sensitive to the relative zeroing of the extensometer and load cell and does not account for the compliance of the resin-rich area at the cut plies. As a result, the absolute delamination length calculated by this equation can be inaccurate and sometimes nonsensical (i.e.: negative values).

These sources of error can be removed by considering the change in delamination length rather than absolute length. By fixing the load, $P$, at which the extensometer strain reading is surveyed, it becomes possible to relate the delamination growth rate to the slope of the extensometer strain plotted vs. the number of fatigue cycles, $N$, by taking the derivative of equation (3) with respect to $N$:

\[
\frac{da}{dN} = \frac{1 - \chi}{\chi} \left( \frac{EBtL_{gauge}}{2P} \right) \frac{d\varepsilon^*}{dN}
\]
Due to the strain energy release rate being independent of delamination length for the central cut-ply specimen, a constant delamination growth rate is expected for constant amplitude fatigue loading. Similarly, according to equation (4), a linear $\varepsilon^* \text{ vs. } N$ behaviour is also expected. This behaviour enables a simple means of calculating delamination growth rates for a single strain energy release rate by applying a linear fit to the extensometer readings of a constant amplitude fatigue test.

### 2.3 Fatigue Tests

Constant amplitude fatigue tests were carried out on central cut-ply specimens fabricated from unidirectional M30SC/DT120 carbon/epoxy prepreg material. The nominal specimen dimensions are shown in Figure 6. Properties of the prepreg material are given in Table 1. The overall laminate consisted of 10-ply of unidirectional material with the two central plies being cut along the entire specimen width. Artificial delaminations were not added during manufacturing. Aluminum tabs were bonded to the ends of the specimens to facilitate gripping in the test frame.

![Cut-ply specimen details](image)

**Figure 6: Cut-ply specimen details.**

| Table 1: Mechanical properties for unidirectional M30SC/DT120 prepreg |
|-----------------|-----------------|-----------------|
| Property        | Manufacturer’s Data | Measured Values |
| $E_{11}$ [GPa]  | 155              | 145 ± 2.5       |
| $E_{22}$ [GPa]  | 7.8              | -               |
| $G_{12}$ [GPa]  | 5.5              | -               |
| $\nu_{12}$      | 0.27             | -               |
| $G_{IIc}$ [kJ/m$^2$] | -              | 1.50 ± 0.05$^*$ |

*Calculated using the measured Young’s modulus*

Fabrication of the specimens began with the production of four 160 mm x 300 mm rectangular carbon/epoxy laminates containing the central cut plies. These panels were cured at a temperature of 120°C and a pressure of 6 bars. Aluminum tabs were secondary-bonded to the rectangular laminates after curing. Specimens with a nominal width of 15 mm were cut from the rectangular panels using a lubricated diamond blade saw. A shorter specimen containing no cut plies was also cut from each panel and used to measure the Young’s modulus of the material using a clip extensometer, which was found to be slightly below the manufacturer’s quoted value (see Table 1).

Static tests were performed in order to determine the critical mode II strain energy release rate, $G_{IIc}$, for the material. Two central cut-ply specimens from each of the four manufactured panels were statically loaded until failure. Prior to static loading, each specimen was pre-fatigued ($R = 0.1$, $G_{IImax} \approx G_{IIc}$) to initiate natural delamination fronts at the four delamination initiation sights. Since the exact length of the initial delaminations were not needed to calculate $G_{IIc}$, they were not measured. However, the delaminations were visually observable in all cases, and thus deemed to be of sufficient length to be free of any boundary condition effects of the cut plié region. Equation (2) was used to estimate $G_{IIc}$ by substituting $P$ with the failure load, defined as the maximum static load observed in the test prior to a
significant drop in load. An average value of 1.50 ± 0.05 kJ/m² was found for $G_{IIc}$ using this approach and the measured value of Young’s modulus.

Table 2 defines the test space and number of specimens for the fatigue test program. The test space focused on the $G_{IImax}/G_{IIc}$ range of 0.7 – 0.9 in order to identify the presence of an asymptotic stable fracture region in the delamination growth behaviour. A single specimen was used for each instance in the test matrix to avoid potential influences of variable amplitude loading on a specimen and to maximize the amount of delamination growth present for each calculation of growth rate. Delamination growth was calculated using equation (4) and the peak strain readings from an MTS 634.12E-24 clip extensometer recorded over the duration of each test. The gauge length of the extensometer was increased to 4” (101.6 mm) using an MTS 634.15B-31 gauge length extender.

Table 2: Fatigue test matrix

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<th>$R$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.75</th>
<th>0.8</th>
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<td>3</td>
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<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>0.5</td>
<td>-</td>
<td>-</td>
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<td>3</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Fatigue tests were carried out on an MTS 810 servohydraulic test frame containing hydraulic wedge grips and a 100 kN load cell (model MTS 661.20E-03). The maximum and minimum forces for each test were calculated using equation (2), the measured specimen geometry, and the measured $G_{IIc}$ value from Table 1. Tests were performed at frequencies varying between 1 – 5 Hz, where lower frequencies were primarily used for tests at $G_{IImax}/G_c = 0.85 – 0.9$ to increase the duration of the test, enabling visual inspections. Previous studies have shown that Mode II delamination growth behaviour for fibre reinforced epoxy materials is not significantly influenced by variations in frequencies in this range.

2.4 Verification of Delamination Growth Measurements

For a select number of tests, optical measurements of delamination lengths were made to verify the assumptions behind equation (4) and validate its accuracy. This equation assumes the delamination state is symmetric across the specimen mid-plane and all four delaminations grow at the same rate.

Regarding these assumptions, Pascoe et al. observed that non-symmetrical disbond growth could occur in thick bonded metal specimens of a similar configuration to the cut-ply specimen. This observed non-symmetry was found to affect the local strain energy release rate, due to secondary bending, at individual disbond fronts in such a way as to promote further non-symmetry. Furthermore, not accounting for the local variation in strain energy release rate resulted in significant scatter in the delamination growth behaviour that masked the true behaviour of the interface.

Despite these observations by Pascoe, the same behaviour was not observed in the present test program using thin specimens. Once initiated, all delaminations grew together and at the same overall rate. Initiation behaviour, however, was not equal for all of the delaminations. It was observed that one pair of delaminations tended to initiate first, resulting in the resin-rich pocket at the cut-ply interface to act an adhesive fillet, delaying the initiation of the second pair of delaminations. This process is illustrated in Figure 7. Delaminations were observed to always initiate in pairs, in adjacent delamination interfaces, growing in the same direction and symmetry in the delamination state across the laminate mid-plane was maintained. As a result, the application of equations (3) and (4) remains valid for this test program.
nominal location of cut in plies

resin-rich pocket

resin-rich pocket

2a

Figure 7: Delayed initiation of second delamination pair due to the formation of a resin-rich adhesive fillet.

These observations are clearly illustrated in a representative extensometer data set shown in Figure 8. Region I of this curve corresponds to the initiation phase for the four delaminations. For this particular instance, the strain begins to linearly increase early in the life, indicating that the first delamination pair initiated very early in the test. The slope of this linear region is half that of Region II, consistent for the behaviour of one delamination pair growing. Within Region II, all four delaminations are growing. The slope of the strain vs. fatigue cycles in this region is used to calculate the delamination growth rate as per equation (4). Finally, in Region III, the delaminations begin to grow beyond the measurement region of the extensometer, resulting in a leveling off of the extensometer reading.

Figure 8: Comparison of extensometer-based and optical-based measurements of crack length and growth for a specimen tested at \( R = 0.1 \) and \( G_{I_{max}}/G_{Ic} = 0.4 \)

A second axis has been added to Figure 8 in order to illustrate the above described issues related to calculating crack length directly from extensometer data using equation (3). Using this approach, a
negative initial delamination length is calculated. Furthermore, calculations of delamination length are
significantly lower compared to optical measurements made for this particular test. However, despite the
error in absolute value of the delamination length, using the slope of the extensometer data combined with
equation (4) to predict delamination growth rate produces good agreement with the optical measurements.

Based on these results, all remaining delamination growth rates in this paper were calculated using
equation (4) using the slope from the linear region II of the extensometer readings from the test. All tests
were run sufficiently long such that either the region III behaviour could be identified in the extensometer
data, or until visual confirmation of the growth of all four delamination could be made. The latter
condition was only used in tests where delamination growth was sufficiently slow that permitting growth
of the delaminations beyond the extensometer gauge length would be prohibitive in testing time (ie: > 3
million cycles).

2.5 Measurement Error Analysis
An often under-discussed topic with respect to the presentation of fatigue delamination growth data is the
influence of measurement error on the results. The parameters of interest, namely delamination growth
rate and strain energy release rate, are seldom measured directly; they are calculated parameters
dependent on several measured quantities. Thus, although individual measurement errors may be small,
their cumulative effects must be considered.

In the present study, the parameters of interest are the strain energy release rate, \( G \), and delamination
growth rate, \( \frac{da}{dN} \), which are calculated using equations (2) and (4) respectively. Consider first the
calculation of strain energy release rate. This parameter is dependent on measurements for specimen
width and thickness, the Young’s modulus of the material, and applied load. The quantity \( \chi \) is exact and
does not have any measurement error associated with it. Thus, in terms of measured quantities:

\[
G_{II} \propto \frac{P^2}{B^2Et} \quad (5)
\]

The relative error, \( \eta \), for \( G_{II} \) can thus be expressed as sum of the relative errors of the measured quantities
as:

\[
\eta_{G_{II}} = 2 \eta_P + 2 \eta_B + \eta_E + \eta_l \quad (6)
\]

where the subscript refers to the quantity the relative error applies to. The relative error is defined by the
absolute error divided by the measured quantity. For instance, the Young’s modulus of the material was
measured as 145 ± 2.5 GPa. The relative error for this quantity is thus 2.5/145, or 1.7%.

The relative errors for \( B, t, \) and \( P \) will not be fixed values as the measured quantities vary from test to test.
All length values were measured using a set of callipers with an accuracy of ±0.05 mm, and the load cell
used during testing had a repeatability of 0.03% of the full range of the load cell, or ±30 N. Using these
quantities, equation (6) can be expressed as:

\[
\eta_{G_{II}} = 2 \left( \frac{30N}{P} \right) + 2 \left( \frac{0.05mm}{B} \right) + 1.7\% + \left( \frac{0.05mm}{t} \right) \quad (7)
\]

For illustrative purposes, using the nominal specimen dimensions from Figure 6 and the load range of 17
– 33 kN, the relative error for \( G_{II} \) is 5.8 – 5.9%. This load range corresponds to the range of maximum
loads applied in the test program. It is clear from this example, although the individual measurement
errors are small, their cumulative effect on the calculated parameter, \( G_{II} \), can be significant.

A similar analysis can be done for the crack growth rate. Following the same approach, the relative error
can be defined as:
\[ \eta_{da/dN} = \eta_{\varepsilon} + \eta_{\varepsilon} + \eta_{\varepsilon_{\text{mop}}} + 2\eta_{\rho} + \eta_{da^*/dN} \]  

(8)

Defining a relative error for the quantity \( \frac{d\varepsilon^*}{dN} \) is not straightforward as this quantity represents a trend in measured quantities. However, the linearity of the curve in Figure 8 (and other similar measurements) suggests that this error is small. Neglecting this term, the relative error for the delamination growth rate is in the same order of magnitude as for \( G_{II} \). The magnitude of this error is much smaller than the scatter in delamination growth rates observed during the test.

Based on the results of this analysis, it was decided that the relative error in \( G_{II} \), as expressed by equation (7), should be indicated on all results using error bars. Conversely, for delamination growth rates, error bars are not included. The scatter observed in repeated tests was found to be more significant than the relative calculation error. One must keep in mind the nature of a log-log scale when visually assessing scatter in the presented plots. A constant measurement accuracy for delamination length would imply a constant scatter band for delamination growth rate (if all scatter can be attributed to this fixed accuracy). However, when plotted on a log scale, this constant scatter band would visually appear larger at the low end of the log-scale.

3 RESULTS

3.1 Onset of the asymptotic stable fracture region

Delamination growth results from the test program are summarized in Figure 9. Delamination growth rates and strain energy release rates were calculated using equations (4) and (2) respectively. As discussed in the previous section, uncertainty due to measurement error is indicated by horizontal error bars for the strain energy release rate and by scatter in the various data points for delamination growth rate. Each data point represents the results from one test specimen according to the test matrix in Table 2.

![Figure 9: Experimental fatigue delamination growth results for MS30SC/DT120.](image)

The delamination growth data in Figure 9 exhibits a non-linear behaviour on a log-log scale. To improve visualisation of this, the data set for \( R = 0.1 \) has been rescaled and plotted in Figure 10. From this figure, the delamination growth data follows the typical linear log-log scale trend for \( G_{II\text{max}} \) below approximately \( 0.7G_{IIc} \). Above this limit, the observed delamination growth rates rapidly increase, deviating from the previous linear trend, consistent with expectations for an asymptotic behaviour. This trend is clearly observed for the \( R = 0.1 \) and \( 0.3 \) data sets, however, it is difficult to ascertain for \( R = 0.5 \) due to the limited number of data points below the \( 0.7G_{IIc} \) limit and the large gradient in delamination growth behaviour below this limit.
Figure 10: Experimental fatigue delamination growth results for $R = 0.1$ rescaled to visualize non-linear behaviour on a log-log scale

It should be noted that there is no theoretical basis for the $0.7G_{IIc}$ limit discussed in this study. It is an observed limit consistent with the data from this study and the data from the literature given in Figure 2 through Figure 4. Its value, however, is plausible in relation to the onset of a stable fracture mechanism. As discussed in \(^1\), the strain energy release rate, $G$, is proportional to the square of the stress intensity factor, $K$, (or square of the applied stress). This is also evident from the proportionality between $G_{II}$ and $P^{\prime}$ in equation (2). Thus, the limit $0.7G_{IIc}$ is equivalent to $0.84K_{IIc}$ or $0.84P_{c}$, where $0.84 = \sqrt{0.7}$. Given the local discontinuities in a composite laminate, it is plausible that localized unstable static fracture could occur this close to the fracture toughness of the material, resulting in accelerated delamination growth and the onset of the asymptotic stable fracture region.

This proportionality between $G$ and $K^2$ also has implications on the definition of a cyclical strain energy release rate range. The cyclical stress intensity factor range, $\Delta K$, typically used to characterize fatigue crack growth in metals, is a measure of the cyclic amplitude of the stress state ahead of a crack tip. It is dependent on the amplitude of this cyclic stress but independent of the mean stress, or in other words, dependent on the cyclic nature of the stress cycle but independent of its monotonic nature. The cyclical strain energy release rate, $\Delta G$, often used as an analogy for $\Delta K$ in delamination growth, does not have this same behaviour due to the above proportionality. From this proportionality, it follows that:

$$\Delta G \propto K_{\text{max}}^2 - K_{\text{min}}^2 = 2(\Delta K)K_{\text{mean}}$$

Thus, $\Delta G$ is dependent on the cyclic nature and mean (or monotonic) nature of the fatigue stress state. A more accurate analogy with $\Delta K$ would be to define the cyclical strain energy release rate range as $(\Delta \sqrt{G})^2$, as is discussed in \(^1\). This discussion is not meant to discredit the potential use of $\Delta G$ as a similitude parameter for characterizing delamination growth, but draw attention to its meaning, and the implication of its use. The authors, however, promote the use of the parameters $(\Delta \sqrt{G})^2$ and $G_{\text{max}}$ as similitude parameters to characterize, respectively, the cyclic and monotonic nature of delamination growth independently.

Figure 11 shows the delamination growth data from this study plotted as a function of $(\Delta \sqrt{G})^2$, with all data points above the $0.7G_{IIc}$ limit highlighted in white. Here, the deviation of the delamination growth from a linear trend on a log-log scale becomes more apparent. Furthermore, the underlying trend for all $R$-ratios appears to be the same when plotted against $(\Delta \sqrt{G})^2$, indicating an absence of mean stress, or monotonic, effects on the delamination growth behaviour. This observation is consistent with previous observations discussed at the beginning of this paper and in \(^1\).
Above the $0.7G_{IIc}$ limit, the delamination growth behaviour does not converge to a single trend when plotted against $(\Delta\sqrt{G_{II}})^2$ (Figure 11) or $G_{II\max}$ (Figure 9). This suggests that both cyclic and monotonic fracture mechanisms are present in this region. Indeed, it is logical that the introduction of local static fracture into the fracture process as $G_{IIc}$ is approached would not completely remove the cyclic fracture mechanism. Curiously, if the delamination growth data from this study is plotted against $\Delta G$, as shown in Figure 12, the data in this region appears to collapse to a single trend. This suggests that the behaviour in this region is related to the specific combination of monotonic and cyclic effects described by equation (9). Further study of this observation is necessary before conclusions about its general validity can be made.

Figure 12: Fatigue delamination growth data plotted as a function of $\Delta G_{II}$

### 3.2 Correlation with existing models

In the previous section, the observed presence of a stable fracture region in the mode II delamination growth curve for the tested material system was discussed. In this section, the implications of this region on the fit of common delamination growth models from the literature will be examined.

The first model examined was proposed by Allegri et al.\textsuperscript{11} as a simplified two-parameter semi-empirical model capable of predicting the influence of R-ratio on mode II delamination growth. This model takes the basic form of a power relation, analogous to the well-known Paris Law, with the normalized...
maximum strain energy release rate, $G_{\text{Imax}}/G_{\text{Ic}}$, as the driving force for delamination growth. R-ratio effects are accounted for in the exponent of the power relation, as shown below:

$$\frac{da}{dN} = C \left( \frac{G_{\text{Imax}}}{G_{\text{Ic}}} \right)^{\frac{b}{1-R}}$$  \hspace{1cm} (10)

Comparison to this model was chosen for illustrative purposes only. The authors acknowledge that Allegri et al. explicitly state that their model extrapolates the so-called Paris Region of the delamination growth curve to the point of static failure, thus neglecting the presence of an asymptotic stable fracture region \(^{11}\). As a result, comparison of the model with the present data can provide an unfair impression of its capabilities. However, the prevalence of fatigue delamination growth models based on extrapolating the Paris Region of the delamination growth curve, particularly without explicit limits on the range of validity of this extrapolation, necessitates discussion with such a comparison. This is meant to draw awareness to implications of having some data within the static asymptotic region on such models, not to criticize such models.

As a regression method for equation (10), Allegri advocates a linear regression of the data plotted on a log-log scale; a log-linear regression. This simplified regression approach is commonly adopted in fitting Paris-type relations for damage growth. Although simple in execution, this regression approach minimizes the squares of the residuals between the logarithms of predicted and observed delamination growth rates rather than minimizing the overall residuals. Practically, this lowers the weighting factors of residuals at large delamination growth rates, relative to those at smaller delamination growth rates, due to the properties of a logarithmic function. As a result, a log-linear regression of equation (10) will tend to generate larger errors at high delamination growth rates compared to a non-linear regression technique.

This potential pitfall is clearly illustrated in the regression of the data set from this study using equation (10) presented in Figure 13. The log-linear regression was performed as outlined in \(^{11}\), while the non-linear regression was carried out using the non-linear bisquare fitting routine available in the MATLAB tool “SFTOOL.” Details of the fitting parameters are given in Table 3. The log-linear regression appears to give a reasonable fit to the data as plotted on a log-log scale; however, the large residuals at the higher delamination growth rates are masked by the log-log scale. The non-linear regression, on the other hand, is heavily influenced by the data points in the region of $G_{\text{Imax}} > 0.7G_{\text{Ic}}$ where the onset of asymptotic behaviour was observed.

![Figure 13: Results of equation (10) fitted to the experimental data set using log-linear and non-linear regression](image)

Which regression technique is more appropriate in this instance is debatable. Indeed, Allegri’s model is not intended to capture the asymptotic behaviour of the delamination growth curve. Thus it could be argued that the weighting effect of a log-linear regression is favourable in this instance. The presence of a
few data points within the asymptotic region of the delamination growth curve would not adversely affect
the apparent fit (i.e. the data sets in Figures 2 through 4). The preferable approach is to identify the limit
that defines the onset of the asymptotic region and constrain the application of the model beyond this
limit. Based on the results from the present study, it could be argued that the literature data presented in
Figure 2 through Figure 4 contain data points already within this asymptotic region that could affect the
fit of non-asymptotic models.

Table 3: Summary of fitting parameters

<table>
<thead>
<tr>
<th>Equation</th>
<th>Log-linear regression $C_{\text{mm/cycle}}$</th>
<th>$b$</th>
<th>Non-linear regression $C_{\text{mm/cycle}}$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10)</td>
<td>0.1895</td>
<td>4.208</td>
<td>0.9834</td>
<td>8.847</td>
</tr>
<tr>
<td>(12)</td>
<td>0.002940</td>
<td>4.121</td>
<td>0.001625</td>
<td>3.122</td>
</tr>
</tbody>
</table>

The second model examined is a variant of the model proposed by Forman et al. 9. In order to account for
the asymptotic stable fracture region and R-ratio effects observed in metal fatigue crack growth, Forman
extended the classical Paris Law as follows:

$$\Delta K = C \left( K_c - K_{\text{max}} \right)$$

(11)

where the factor of $(1 - R)$ in the denominator accounted for R-ratio effects and the factor $(K_c - K_{\text{max}})$
introduced the asymptotic behaviour in the stable fracture region. $C$ and $b$ are material dependent
parameters.

In order to apply this equation to mode II delamination growth data in this study, the stress intensity
factor values $(K_c - K_{\text{max}})$ and $\Delta K$ are substituted with $(\sqrt{G_{\text{IIc}} - \sqrt{G_{\text{II max}}})^2}$ and $(\Delta \sqrt{G_{\text{II}}})^2$ respectively.
Additionally, the term $(1 - R)$ is dropped due to the absence of R-ratio effects for mode II delamination as
discussed in 10 and as demonstrated for this data set in Figure 11. The resulting modified Forman relation
is thus given by:

$$\frac{da}{dN} = \frac{C (\Delta \sqrt{G_{\text{II}}})^{2b}}{(\sqrt{G_{\text{IIc}} - \sqrt{G_{\text{II max}}})^2}}$$

(12)

where the material dependent parameter $C$ and $b$ are different than those in equation (11).

Results from the regression of the data from this study using equation (12) are plotted in Figure 14. As
with the previous model, log-linear and non-linear regression analyses were performed. Details of the
fitting parameters from these analyses are given in Table 3. The closer agreement between the two
regression analyses for this model highlights its ability to capture the observed behaviour at higher
delamination growth rates; the effect of reducing the weighting of residuals for the higher delamination
growth rates in the log-linear regression is minimal. Furthermore, both regression analyses produce a
good fit to the experimental data.
Figure 14: Results of equation (12) fitted to the experimental data set using log-linear and non-linear regression

The authors contend that the form of this model is more appropriate for characterizing mode II delamination growth than other models based on maximum strain energy release rate. Use of $(\Delta \sqrt{G_{II}})^2$ as a similitude parameter for characterizing fatigue delamination growth correlates well with experimental observations and links the damage process to the amplitude of the fatigue stress state at the damage; a cyclic loading process. Deviations from this behaviour can be explained by the onset of a stable fracture mode, resulting in an asymptote as the fracture toughness of the material is approached; a monotonic loading process. Although equation (12) captures both of these processes, it still relies on material dependent parameters lacking any known physical meaning. Furthermore, there is no physical basis for the intensity of the asymptote gradient imposed by the $1/(\sqrt{G_{IIc}} - \sqrt{G_{IImax}})^2$ factor or its implied onset. These shortcomings may be acceptable for the purposes of an engineering model, but solutions to them should be sought and their limitations understood.

4 CONCLUSIONS

A series of fatigue tests have been performed to investigate the onset of the asymptotic stable fracture region in the Mode II fatigue delamination growth behaviour of a unidirectional carbon/epoxy material. Based upon the results of these tests, the following conclusions can be made:

- An asymptote in the delamination growth rate vs. maximum strain energy release rate behaviour of the material system under investigation was observed. This asymptote was most evident in the data sets obtained at fatigue stress ratios of 0.1 and 0.3.

- The onset of the asymptotic region appears to occur at approximately $0.7G_{IIc}$. Based on the strain energy release rate being proportional to the square of the stress intensity factor, this correlates with $0.84K_{IIc}$. This represents a plausible value for the onset of localized stable static delamination growth under fatigue loading given the local discontinuities within a composite laminate.

- Mode II fatigue delamination growth behaviour below the observed asymptote shows a good correlation with $(\Delta \sqrt{G})^2$, consistent with previous findings of the authors and consistent with other studies utilizing an analogous stress intensity factor description, $\Delta K$.

Based upon these results, existing Mode II fatigue delamination growth models from the literature were fitted to the data obtained in this study to investigate the impact and potential dangers of ignoring this asymptotic behaviour on delamination growth predictions. The data from the study was found to correlate well with a modified Forman equation based on the classical Paris Law with $(\Delta \sqrt{G})^2$ as the similitude parameter for damage growth and a correction factor of $1/(\sqrt{G_{IIc}} - \sqrt{G_{IImax}})^2$ to define the onset of the asymptotic stable fracture region. Although good correlation was observed, it is noted that there is no
physical basis for $1/(\sqrt{G_{IIc}} - \sqrt{G_{IImax}})^2$ defining the onset of this region, thus its applicability for other material systems needs to be investigated further.

Finally, this study limited itself to positive R-ratios. For negative R-ratios, it is possible for the critical energy release rate to be approached by both the maximum and minimum portions of the fatigue load cycle. This could result in a variation in the apparent onset and/or strength of the asymptote, however, an in-depth investigation into this is required. Usage of the cyclic strain energy release rate range ($\Delta \sqrt{G}$)$^2$ for negative R-ratios is further discussed in 10.

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