Fatigue Delamination Growth for an Adhesively-Bonded Composite Joint under Mode I Loading

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Abstract: This paper focuses on experimental study and modelling of load ratio effects on fatigue delamination growth of an adhesively-bonded composite joint under Mode I loading. Owing to the development of an in-house Automatic Delamination Monitoring System (ADMS), images of the delamination front location can be captured in a continuous, non-contact manner in synchronization with the peak load of the hydraulic test frame. This has led to enhanced data quality and consistency. The test results show that fatigue delamination growth in the steady delamination growth range, up to the point when \( G_{\text{max}} \) reaches the critical strain energy release rate (\( G_c \)), follows the Paris relationship with excellent repeatability at displacement ratios of 0.1, 0.3, 0.5, and 0.7. A “subcritical fatigue growth” knee region, historically observed in metal fatigue crack growth curves and some composite delamination growth curves whereby a fatigue threshold value can be established, was not observed in this study.

A fatigue delamination model was developed by characterizing the damage zone near the delamination tip and the relationship between the damage accumulation rate in the damage zone and the delamination growth rate. A linear constant life diagram for composite failure under in-plane loading was applied to extract the form for effective stress in the damage zone assuming that matrix cracking-dominated material behaviour is not affected by the type of loading. The developed delamination growth model, taking into account both static and cyclic fatigue delamination, has demonstrated a reasonable fit to experimental data for the \( R \)-ratio effect on the delamination growth rate of a bonded composite joint under Mode I loading. Comparison between the experimental data and the developed model, as well as several others was conducted. It was shown that among the models characterized by a simple power relationship with \( R \)-ratio effect characterized by the base of the equation, the developed model provided the best fit. Another type of delamination growth model, which features an \( R \)-ratio defined in the power exponent, may provide improved predictions in that it is able to capture all observed trends in the experimental data.

INTRODUCTION

One of the major advancements of the 20\textsuperscript{th} century is the usage of composite materials for a wide range of applications, including aerospace, automotive and construction. In the aerospace industry, usage of composite materials not only helps meet industrial needs for lightweight structures driven by high fuel cost and increasingly stringent regulation for air and noise pollution, it also offers advantages of fatigue and corrosion resistance and the capacity for manufacturing large, complex structures. As a result, composite materials have been increasingly used for structural applications, where fatigue loading is present and dissimilar materials are often integrated, such as helicopter blades. Although experience with composite applications has been mostly positive [1], composites are susceptible to delamination and disbond. The sources of delamination and disbond may arise from manufacturing defects, impact damage and environmental degradation [2]-[4]. Delamination and disbond is one of the major sources of part failure because damage that exceeds the threshold may render little visual indication and, once the damage dimension exceeds the threshold, it may propagate at small load, leading to catastrophic failure [2]. An example is the F-18 composite structures where delamination and disbond was the major source of concern [4]. When not detected early, the disbond in the control surfaces had led to reduced stiffness and strength and the eventual in-flight failures [4].
A fracture mechanics approach to crack growth was introduced to aircraft design after a F-111 accident [5] following early successful applications of fracture mechanics to investigate the fatigue failure such as failure of several comet jet aircraft [6]. This approach has been widely used for a range of studies of delamination growth behaviour in composite structures. When designing composite structures, there are two major design philosophies for the treatment of potential in-service delamination and disbond damage. In the no-growth design philosophy, it must be demonstrated that potential in-service damage will not grow (either statically or cyclically) under service loading conditions. Thus, it is necessary to understand the limits for static growth and thresholds for fatigue growth of such damage. In the traditional damage tolerance approach applied in metallic structures, a slow-growth and detection philosophy is adopted. In such cases, damage growth is predicted and inspection intervals are determined to detect the damage before it reaches a certain value. Thus, a thorough understanding of the limits of static growth and the fatigue growth behaviour are necessary. For both philosophies, the application of fracture mechanics can help provide needed understanding.

Due to the difficulty of obtaining the local delamination tip stress field for the determination of the stress intensity factor, $K$, in inhomogeneous composite laminate, the strain energy release rate, $G$, has been used [7]. Within this energy-based approach, failure occurs when the energy available for damage growth exceeds the resistance of the material. Strain energy release rate, one form of fracture toughness is defined to be the loss of potential energy, $dU$, in the test specimen per unit of specimen width for an infinitesimal increase in delamination length, $da$, for a delamination growing under a constant displacement (see Eqn.(1)).

$$ G = -\frac{1}{b} \frac{dU}{da} $$  \hspace{1cm} (1)

where, $b$ is the width of the specimen. Analogous to the power relationship commonly used to describe the dependency of crack growth rate in metals on the stress intensity factor, $K$ or $\Delta K$, the fatigue delamination growth rate, $da/dN$, of a fibre reinforced composite for a given $R$-ratio can be captured by a power function of the maximum strain energy release rate, $G_{\text{max}}$, as shown in Eqn.(2) [7][8].

$$ \frac{da}{dN} = C(G_{\text{max}})^n $$  \hspace{1cm} (2)

where, $C$ and $n$ are material parameters.

The effect of stress ratio on delamination growth rate of composite materials under various loadings has been widely studied [9]-[14]. Variations in stress ratio result in a variation of the mean cyclic stress. Thus, a stress ratio dependency on delamination growth behaviour indicates the presence of cyclical (stress amplitude) as well as monotonic (mean stress or maximum stress) effects on the growth behaviour.

Mall [9] had success using $\Delta G$ as the driving parameter for cyclic debonding of adhesively-bonded joints made from T300/5208 adherends bonded with EC 343 adhesive when subjected to Mode I and mixed Mode I and II cyclic loading with different stress ratios. The usage of $\Delta G$, Mall proposed, took into account the $R$-ratio effect in characterizing the mechanics of cyclic disbonding because $\Delta G = G_{\text{max}} - G_{\text{min}}\sim (K_{\text{max}}^2 - K_{\text{min}}^2) \sim (1 + R)\Delta KK_{\text{max}}$. This approach was considered to be consistent with the manner in which fatigue crack growth rates in metals are expressed using any of the two following parameters, namely, $K_{\text{max}}$, $K_{\text{min}}$, $K_{\text{mean}}$, $\Delta K$, and $R$. Thus, the power-law relation for cyclic debond growth as in Eqn.(2) was proposed to account for the stress ratio effect:

$$ \frac{da}{dN} = C(\Delta G)^n $$  \hspace{1cm} (3)

where, $C$ and $n$ are material parameters. Although relatively good success was obtained by Mall in using $\Delta G$ as a driving parameter for delamination growth, no reasoning for the relative contribution of monotonic and cyclical effects dictated by this parameter were given.

Based on the Walker parameter developed for fatigue behaviour of metals [15], Hojo [11][12] introduced a constant, $\gamma$, to account for the relative contribution of the maximum and the cyclic stresses on the delamination growth rate of unidirectional laminates made from two types of prepreg, T300/914C and Toray P305, under Mode I loading (see Eqn.(4)). The higher value of $\gamma$ for T300/914C laminate suggests that the mechanism of fatigue delamination propagation is mainly controlled by static fracture, while the lower $\gamma$ value for Toray P305 indicates an increased effect of cyclic stress on delamination growth.
\[ \frac{da}{dN} = C(\Delta K^{1-\gamma}K_{\text{max}}^\gamma)^n \]  

(4)

where, \( \gamma \) is a constant representing the relative contribution of static and cyclic stresses to delamination growth.

Anderson [13] developed a fatigue delamination model of composite laminates under Mode I or Mode II loading based on the assumption that it is the damage accumulation ahead of the delamination tip that drives delamination propagation, and a linear accumulative assumption within the damage zone was applied. For Mode I fatigue loading, the delamination model is given by Eqn.(5):

\[ \frac{da}{dN} = C \left( \frac{\Delta K_I - \Delta K_{\text{th0}}(1 - R)c_1 + c_2/\sqrt{K_{\text{ic}}}}{K_{\text{ic}} - \Delta K_{\text{in}}^\ell} \right)^b \]  

(5)

where, \( c_1, c_2 \) and \( b \) are empirically-derived constants. \( K_{\text{ic}} \) and \( G_{\text{ic}} \) are Mode I critical stress intensity factor and critical strain energy release rate, respectively. \( \Delta K_{\text{th0}} \) represents the threshold mode I stress intensity factor range below which fatigue would not grow. \( \Delta K_I \) and \( \Delta K_{\text{in}}^\ell \) are the range of Mode I stress intensity factor and the mean value, respectively.

If the models can be summarized in the basic form of \( \frac{da}{dN} = C(K_{\text{eff}})^b \), the majority of them take into account the \( R \)-ratio effect by defining \( K_{\text{eff}} = K_{\text{eff}}(R) \). Allegri [14] proposed a semi-empirical model for stress ratio effect for Mode II fatigue delamination growth of composite laminates by defining \( b = b(f) \) as shown in Eqn (6).

\[ \frac{da}{dN} = C \left( \frac{G_{\text{ic,max}}}{G_{\text{ic}}} \right)^{\frac{b}{(1-R)^m}} \]  

(6)

where \( G_{\text{ic,max}} \) is the Mode II toughness and \( C \) and \( b \) are empirically-derived constants. Although this mode was developed for Mode II delamination growth of composite laminates, this form of equation provides an interesting alternative to considering the effect of \( R \)-ratio.

While a vast majority of the delamination rate test results in the aforementioned work were found to follow a power law relationship to \( K \) or \( G \) for any given \( R \)-ratio, an effort was made by Shahverdi [16] to develop a fatigue life model of bonded composite joints under Mode I loading to capture the three regions of delamination growth, namely, subcritical around fatigue threshold, the steady region controlled by \( G_{\text{ic}} \), and the critical region close to \( G_{\text{ic}} \) as shown in Eqn (7). This model is based on the total fatigue mode proposed by Shivakumar [17] for composite delamination growth rate at \( R=0.1 \).

\[ \frac{da}{dN} = D(R)(G_{\text{ic,max}})^m(R) \left( 1 - \frac{G_{\text{ic}}(R)}{G_{\text{ic,max}}} q_1 \right) \left( 1 - \frac{G_{\text{ic}}(R)}{G_{\text{ic}}} q_2 \right) \]  

(7)

where \( D(R), m, \) and \( G_{\text{ic}}(R) \) are a function of \( R \)-ratio. \( q_1 \) and \( q_2 \) are constants.

What is worth mentioning is that while the linear elastic parameter \( \Delta G = G_{\text{max}} - G_{\text{min}} \) was used by multiple researchers to characterize the fatigue delamination growth behaviour [11][18][19], the use of such definition of \( \Delta G \), as pointed out by Rans [20], may result in misinterpretations of delamination growth data when \( \Delta G \) is used as the basis for similitude of the cyclic load range. Instead, a definition of \( \Delta G = (\sqrt{G_{\text{max}}} - \sqrt{G_{\text{min}}})^2 \) would make the analysis of delamination data become analogous to using \( \Delta K \), leading to improved prediction of delamination data from literature [11][18].

In this study, a fatigue delamination model for adhesively-bonded composite joints was developed based on the approach proposed by Bolotin [21][22] and Anderson [13], where the delamination growth is driven by fatigue damage accumulation in the damage zone near the delamination tip.
3.1 Specimen design and fabrication

A modification to the double cantilever beam specimen design as per ASTM D 5528-1 [23] was made to conduct fracture toughness tests of an adhesively-bonded joint configuration. The bonded composite specimens were made with two 13-ply of unidirectional continuous carbon fibre-reinforced composite material, Cytec CYCOM 5276-1, bonded with 3M AF163-2K adhesive as shown in Fig. 1. The unidirectional laminates, with an average thickness of 1.8mm (0.072"), were cured following the cure cycle recommended by the manufacturer. Grit blast surface preparation was conducted on the surfaces of the laminates prior to bonding. Two plies of adhesive with a nominal thickness of 0.24mm (0.0095") per ply were used. A 0.013mm (0.0005") thick Teflon insert was placed between the plies of adhesive at one end of the specimen. A two-step cure cycle was applied, with the first at 95°C and 345 kPa for 20 minutes and the second at 121°C and 345 kPa for 70 minutes. The first step was to allow the excess adhesive to flow out and reduce the overall bondline thickness to values more representative of real applications, where 0.13–0.26 mm thick bondlines are often considered optimal [24]. Each panel was given a batch letter and panel number based on the bonding step (e.g. C01–C04 were four different panels bonded at the same time). The bonded panels were cut and hinges were bonded to them with Hysol EA 9360 paste adhesive, as shown in Fig. 1. The final bondline thickness of the test specimens was 0.33 ± 0.07 mm and a loading point is 50.8 mm (2") away from the the centre of the hinge ($a_0$) (see Fig. 1).

![Fig. 1: Schematics of the Adhesively-Bonded Composite Specimen](image)

Experimental set-up

All experiments were conducted in laboratory conditions, at a temperature and humidity level of 22±5°C and 50±20%, respectively. All the tests were conduct under displacement control at a frequency between 0.5 to 2.5 Hz. The test utilized a MTS 858 Tabletop hydraulic test frame, using a 250N or a 500N load cell with 1% load accuracy. The data acquisition station used TestStar II Station Manager v3.4B to record the load and crosshead displacement during testing.

The specimens were inserted in grips in the frame by aligning and securing the lower hinges first followed by gripping and tightening the upper hinge on the specimen. The load and displacement were zeroed prior to test. Once the specimen was installed, a loading ramp according to ASTM D 5528-1 was applied to the grips. Once the displacement reached the target maximum value, fatigue loading was applied to the specimen at a set frequency, typically 1 Hz. Time, cycle count, load and displacement data were collected by the load frame control software. A reduced set of information was also captured by the Labview control software, along with the images of delamination tip location at the peak of the cycles.
Automatic delamination monitoring system (ADMS)

The calculation of strain energy release rate, $G$, is based on the measurement of specimen compliance and delamination length. While specimen compliance can be directly obtained from load frame measurements, it is not an easy task to measure delaminate length during a test. Several methods were explored by various studies. Shahverdi [25] compared four methods for determining the delamination length including two direct methods - visual observation (involving interruption of test and measurement of the length using optical microscope) and crack gauge, and two indirect methods – dynamic compliance calibration [26] and quasi-static compliance calibration. Compliance calibration methods estimate the average delamination length using measured compliance based either on measured compliance change of a specimen during fatigue or a quasi-static loading. The visual method is onerous to conduct, requiring interruption of tests and often yielding limited data points. Crack gauges measure delamination length by measuring electrical resistance as the set of wires in the gauge is progressively cut. The drawbacks of this method are high cost, the limited number of measurement points determined by the density of wires, and the contact between the gauge and the specimen. Compliance methods were found to be unsuitable for the purpose of delamination length calibration because considerable scatter in the calibration test made adoption of a single compliance for all cases difficult [25].

An automatic delamination monitoring system (ADMS) was developed in this study to measure delamination growth directly in a continuous manner. This was conducted by acquiring images of delamination tip positions in synchronization with load frame reading during a fatigue test, or acquiring images along with load or displacement reading from the frame during a static test. The camera system used for the study was capable of taking images up to 40 frames per second, and the image acquisition frequency can be tailored to match the rate of delamination growth. The ADMS consists of a digital camera, a 3-axis travelling base, and a Labview based-control system to analyse frame signals and trigger the camera operation. These images (see Fig. 2 for an sample) were manually examined in post-test analysis using UTHSCA ImageTool version 3.0. This non-contact system allows for delamination length measurement as frequent as desired in an uninterrupted manner, ensuring maximum data quality and accuracy. The flexibility the system offers in terms of data density and potential for real-time determination of delamination length for test control make it an attractive technique for crack/delamination monitoring for both metallic and composite fatigue testing.

![Fig. 2: A Sample Image Acquired from the Automatic Delamination Monitoring System (ADMS)](image)

Test Matrix

The Mode I critical strain energy release rate of bonded joints, $G_{IC}$, was first measured in static tests with a displacement rate of 2.0 mm/min, following the procedure described in ASTM D5528[23]. The set maximum cyclic displacement value for the fatigue tests was calculated based on the measured critical displacement at the same delamination length, $\delta_{IC}$, and $G_{IC}$ obtained from quasi-static tests, as well as the target maximum strain energy release rate corresponding to the maximum load in a cycle, $G_{imax}$, using the relationship $G_{imax}/G_{IC} = (\delta_{imax}/\delta_{IC})^2$. Due to the displacement control mode used for the test, $R$-ratio in this study is the displacement
ratio, $\delta_{\text{min}}/\delta_{\text{max}}$. Assuming delamination growth within a cycle is negligible, the compliance $C = \delta_{\text{max}}/P_{\text{max}} = \delta_{\text{min}}/P_{\text{min}}$ of the double cantilever beam is constant within a cycle and the load ratio of the Mode I fatigue test ($P_{\text{min}}/P_{\text{max}}$) is the same as the displacement ratio.

The effects of $R$-ratios of 0.1, 0.3, 0.5 and 0.7 on delamination growth were evaluated to cover a range of fatigue loading with low to high $R$ values. For each $R$-ratio, tests were performed on each specimen to cover high to low load static fatigue, typically between 0.2 $G_{\text{IC}}$ and 1.0 $G_{\text{IC}}$. Five or six specimens were tested at each $R$-ratio to ensure repeatability. Table I shows the test matrix for the fatigue tests.

Table I. Summary of Mode I Fatigue Fracture Toughness Tests for the Study of the $R$-Ratio Effect

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Type</th>
<th>$R$-ratio</th>
<th>Target $G_{\text{IC}}$</th>
<th>Total No. of Cycle</th>
<th>No. of Delam. Location Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>Fatigue</td>
<td>0.1</td>
<td>$1.2 G_{\text{IC}}$</td>
<td>70,000-930,000</td>
<td>3,000-5,000</td>
</tr>
<tr>
<td>6-11</td>
<td>Fatigue</td>
<td>0.3</td>
<td>$1.2 G_{\text{IC}}$</td>
<td>70,000-340,000</td>
<td>2,000-5,000</td>
</tr>
<tr>
<td>12-16</td>
<td>Fatigue</td>
<td>0.5</td>
<td>$1 - 1.2 G_{\text{IC}}$</td>
<td>75,000-92,000</td>
<td>1,000-2,600</td>
</tr>
<tr>
<td>17-21</td>
<td>Fatigue</td>
<td>0.7</td>
<td>$G_{\text{IC}}$</td>
<td>84,000-103,000</td>
<td>900-2,200</td>
</tr>
</tbody>
</table>

Experimental Results and Discussion

4.1 Onset and Propagation of $G_{\text{IC}}$

The critical energy release rate, $G_{\text{IC}}$, represents the threshold of resistance to delamination growth under Mode I quasi-static loading. The values of onset and propagation strain energy release rates were generated in a static test, where pre-cracking was performed following ASTM D5528 [23] to remove the effect of the Teflon insert. A follow-on failure mode analysis showed that all specimens failed cohesively (the failure occurred in the adhesive layer) according to ASTM D 5573-99 [27].

Three methods as described in ASTM D5528 [23] were used to determine the onset strain energy release rate, $G_{\text{IC}}$, including the visual onset of delamination growth, the beginning of non-linearity of the load-displacement curve ("NL") and the 5% offset/max load where the 5% offset was defined to be the intersection of the curve with 5% increase in compliance from the linear portion of the $P$–$\delta$ curve with this curve. The propagation strain energy release rate $G_{\text{IC}}$ was calculated using three methods including Modified Beam Theory (MBT), compliance Calibration (MC) and Modified Compliance Calibration (MCC).

A typical Mode I fracture toughness result is shown in Fig. 3, where propagation $G_{\text{IC}}$, calculated from MBT, CC and MCC methods are shown to be virtually the same. The analysed results also demonstrated that onset $G_{\text{IC}}$ determined from methods of visual observation of delamination onset and 5% offset/max load also gave the same value, while the NL method offered a conservative and in many cases an unreliable value of Onset $G_{\text{IC}}$ due to inconsistency of NL point determination from person to person. It can be noted that not only did propagation $G_{\text{IC}}$ remain fairly constant for the test region of 50 mm delamination propagation length, for these adhesively-bonded joints, the onset $G_{\text{IC}}$ values were also the same as propagation $G_{\text{IC}}$. This observation indicates that, unlike delamination in a composite laminate where fibre bridging plays a role in fracture toughness measurement, fracture toughness of bonded joints is primarily determined by the adhesive material behaviour provided that the cohesive failure occurs. This observed consistency of onset and propagation of $G_{\text{IC}}$ also yields a significant assumption for studying fatigue behaviour of composites.

Although the measured $G_{\text{IC}}$ values for these adhesively-bonded joints are relatively consistent within a batch, an apparent batch-to-batch difference in the average $G_{\text{IC}}$ value was observed. All specimens were made and tested following the same protocols and the cohesive failure mode was observed for all specimens, but $G_{\text{IC}}$ values of composite joints bonded with AF163-2K was measured to be in the range of 2800 – 4000 J/m². Therefore, it is highly recommended that static tests be performed on each batch of the specimens, wherever possible.
Calculation of strain energy release and delamination growth rate

The maximum cyclic strain energy release rates of the Mode I DCB joints were calculated based on linear elastic fracture mechanics [23].

\[
G_{IC} = \frac{3P\delta}{2b(a + |\Delta|)}
\]

where the correction factor \(|\Delta|\) was applied to take into account the rotation at the delamination tip and the moment resulting from large displacements [27].

The secant method and 7-point polynomial method are typically used to calculate the delamination growth rate, according to ASTM E-647-08. The secant method, or point-to-point method, determines the delamination rate by calculating the slope of a straight line connecting two adjacent data points on the \(a-N\) curve. The 7-point polynomial method fits a second-order polynomial equation to sets of 7 successive data points and the delamination growth rate is determined by the slope at any point along the line. The 7-point or incremental polynomial fitting method requires that the \(N\) interval be the same between all data points. However, there is a varying cycle number increment for the delamination length measurement. Thus, the secant method was used to calculate the delamination growth rate.

Fatigue delamination growth curves

All fatigue tests were conducted at 1 Hz in a continuous manner, starting from \(G_{imax}\) as high as \(G_{IC}\) until \(G_{imax}/G_{IC}\) reached close to 0.2 at which delamination growth rate had significantly reduced. The only exception to this test protocol was a set of 3 fatigue tests at an \(R\)-ratio of 0.1, where the test sequence was designed to overcome the relatively large displacement of the crossheads required for DCB testing, and the limitation of the 14.7 KN (3.3 Kips) hydraulic frame to maintain sinusoidal loads. The revised test sequence on an as-made DCB specimen began with fatigue cycling aiming at a low target of \(G_{imax}\) of 0.3 and a frequency at 2.5 Hz, followed by an increase in cyclic displacement to cover the fast growth region and steady regions of the curve at a frequency of 1 Hz. As shown in Fig. 4, the two test protocols gave the identical fatigue delamination growth curves in spite of the fact that the three tests were conducted continuously under 1 Hz using one set of displacement value, and the other three conducted in a segmented manner with varying frequencies and multiple displacement values to reach the desired \(G_{imax}\) for each segment. These tests with the \(R\)-ratio of 0.1 showed that, within the test range, the delamination growth rate followed a power law relationship and no delamination threshold was observed.
**R-ratio Effect on delamination growth curves**

The Mode I fatigue delamination growth curves of adhesively-bonded composite laminates at R-ratios of 0.1, 0.3, 0.5 and 0.7 are shown in Fig. 5. Several trends in delamination growth with change in maximum strain energy release rate, \( G_{imax} \) and R-ratio can be observed. First, each of the curves were found to follow a power relationship between delamination rate and the maximum strain energy release rate. A higher \( G_{imax} \) (or maximum cyclic stress) led to a faster delamination growth rate. Second, delamination growth rate is shown to be highly dependent on the R-ratio. In general, for any given \( G_{imax} \) (or maximum cyclic stress), a lower R-ratio (or a higher cyclic stress range) led to a faster growth rate. This indicates that there are two essential driving forces for delamination growth under Mode I loading, namely, \( G_{imax} \) (or \( G_{im} \), the mean value of \( G_i \) in a fatigue cycle) and \( \Delta G_f \). Third, the delamination growth rates for all R-ratios were found to be fairly close to each other when the maximum energy release rate reached the critical value (\( G_{imax} = G_{IC} \)). This point represents a quasi-static delamination growth phenomenon, independent of the R-ratio.

![Fig. 5: Experimental Results of R-ratio Effects on Fatigue Delamination Growth of Bonded Composite Joints](image)

**Model Development of Fatigue Delamination Growth**

**Interlaminar Fatigue Damage Accumulation and Delamination growth**

Interlaminar damage accumulation has been introduced to model interlaminar delamination propagation by several studies [21][22][28][32]. Lee [28][29] related the interlaminar fracture toughness of brittle-matrix laminates to the Mode II microcracking pattern formation and coalescence observed at the delamination tip. Allix [30][32] modelled delamination onset and propagation under quasi-static loading by linking it to interlaminar damage accumulation. Anderson [13] has explicitly introduced this concept to model Mode I delamination of a composite laminate to study stress effects on delamination rate. In this work, the approach elaborated in [13][21][22] is used.

Interlaminar accumulated damage defines damage, \( D \), ahead of the delamination front under cyclic interlaminar stresses. The delamination grows once the accumulated damage \( D \) reaches a critical level, \( D_C \). For steady growth, the damage zone ahead of the delamination front is characterized by a length of \( \rho \), as shown schematically in Fig. 6. Assuming the damage zone is sufficiently small, the delamination growth rate, \( da/dN \), can be assume to be constant...
over the length of damage zone, \( \rho \). Thus the number of cycles, \( \Delta N_c \), for delamination to grow to cover the damage zone is expressed in Eqn (9).

\[
\Delta N_c = \frac{\rho}{d\alpha/dN} \quad (9)
\]

The rate of damage accumulation rate, \( \partial D/\partial N \), depends on the static, \( \sigma_{\text{max}} \), and cyclic, \( \Delta \sigma \), interlaminar stresses:

\[
\frac{\partial D}{\partial N} = f(\sigma_{\text{max}}, \Delta \sigma) \quad (10)
\]

For steady growth, when the damage zone is sufficiently small, the interlaminar stresses within the zone are assumed to be constant until the delamination grows to cover the damage zone, \( \rho \). Thus, the Eqn.(10) can be rewritten into Eqn. (11).

\[
D = D_0 + f(\sigma_{\text{max}}, \Delta \sigma) \times \Delta N \quad (11)
\]

The number of cycles for the damage within the zone to exceed the critical value, \( D_c \), is then calculated using Eqn.(12).

\[
\Delta N = \frac{(D_c - D_0)}{f(\sigma_{\text{max}}, \Delta \sigma)} \quad (12)
\]

where \( D_0 \) is the pre-existing damage in the damage zone. Combining Eqn (9), (10) and (12), delamination growth can be expressed by Eqn.(13).

\[
\frac{da}{dN} = \frac{\rho}{(D_c - D_0)} f(\sigma_{\text{max}}, \Delta \sigma) = \frac{\rho}{(D_c - D_0)} \frac{\partial D}{\partial N} \quad (13)
\]

Eqn.(13) shows that fatigue delamination growth rate is proportional to the damage accumulation rate ahead of the delamination front. The benefit of linking the delamination growth rate with the damage accumulation rate is that there have been a significant amount of empirical data generated for damage accumulation and constant life diagrams for composite laminates.

Constant life diagrams, first proposed in the late 19th century [33][34], are graphical representations of the safe regimes of loading for a given specific life, e.g. the endurance limit for infinite life. Multiple researchers have used constant life diagrams to characterize composite fatigue [35][36]. Philippidis [35] developed constant life diagrams to illustrate the life of composite laminate, made of various ply angles under complex in-plane stress states.
Sarfaraz [36] examined the axial, tensile, compressive and reversed fatigue behaviours of a double lap joint made of pultruded glass fibre reinforced composite laminates bonded by an epoxy adhesive system. Kawai [37] constructed a constant fatigue life diagram for composite laminates of various layups under tension-tension, tension-compression, compression-compression loading. The data from these studies demonstrated that, within the R-ratio range of 0 to 1, a linear constant life diagram can be used to characterize composite laminates under in-plane stress states. It is conceivable that the fatigue life of the matrix phase in a composite or adhesive in a bonded joint follow a similar diagram whether the material is subjected to in-plane or out-of-plane loading. A linear constant life diagram is proposed in this study for the fatigue damage behavior in the damage zone near the delamination tip of the adhesively-bonded composite joints under out-of-plane loadings (including Mode I, II and mixed Mode), as shown in Fig. 8. In this diagram, each constant life curve \( N_i \), defines the relationship between static and cyclic interlaminar stresses, \( \sigma_{\text{max}} \) and \( \Delta \sigma \) to reach the specific life.

As illustrated in Fig. 8, when \( R = 1 \), static stress, \( \sigma_{\text{max}} \), reaches the interlaminar strength of the DCB specimens, \( \sigma_c \). When \( \sigma_{\text{mean}} = 0 \), \( R = -1 \) and an endurance limit of \( \Delta \sigma_{-1} \) can be set for all in-plane and out-of-plane loading cases except for pure Mode I. Under Mode I loading, the valid range for the constant life diagram is \( R = 0 \sim 1 \). When a DCB specimen is subjected to tension-compression loading (\( R < 0 \)), the fatigue delamination growth rate is essentially the same as the case at \( R = 0 \) due to crack closure in compression.

![Fig. 8: Proposed Linear Constant Fatigue Life Diagram for Adhesively-Bonded Composite Materials. Mode I Fatigue Falls in the Shaded Area Only.](image)

The effective stress, \( \sigma_{\text{eff}} \), for the region of the constant life diagram based on Fig. 8 can be defined in Eqn.(14):

\[
\sigma_{\text{eff}} = \frac{\Delta \sigma}{(\sigma_c - \sigma_m)}
\] (14)

Assuming a linear damage accumulation rate according to Palmgren and Miner and a simple power dependence of fatigue life on the effective stress, one can obtain the following relationship:

\[
\frac{\partial D}{\partial N} = C_D \left( \frac{\Delta \sigma}{(\sigma_c - \sigma_m)} \right)^b
\] (15)

Combining Eqn.(13) and Eqn.(15), the delamination growth model for the study of the fatigue behaviour of the adhesively-bonded joint can be obtained as shown in Eqn.(16).

\[
\frac{da}{dN} = C \left( \frac{\Delta \sigma}{(\sigma_c - \sigma_m)} \right)^b
\] (16)

where, \( C, C_D \) and \( b \) are damage accumulation constants. \( C = \frac{\rho c_D}{(\sigma_c - \sigma_0)} \)
For brittle-matrix laminates, plasticity effects can be neglected, and the stress within the damage zone can be taken proportional to the stress intensity factor, $K$ [13]. For plane strain conditions, the relationship between stress intensity factor, $K$, and strain energy release rate, $G$, can be expressed in Eqn.(18) when failure occurs cohesively. However, should failure occur in the adherend (or the laminate), Eqn.(19) is used to describe the relationship between the fracture toughness of the laminate under plane strain conditions [38].

\[ \frac{\sigma}{\sigma_c} = \frac{K}{K_c} \]  

(17)

\[ G = \frac{K^2}{E_a} = \frac{(1 - v^2)}{E} K^2 \]  

(18)

\[ G = \frac{K^2}{E_l} = K^2 \times \sqrt{\frac{(1 - v_{13}v_{31})(1 - v_{23}^2)}{2E_1E_2}} \times \left( \frac{E_1(1 - v_{23}^2)}{E_2(1 - v_{13}v_{31})} + \frac{1}{2}\frac{G_{12} - 2v_{23}(1 + v_{23})/E_2}{(1 - v_{13}v_{31})/E_1} \right)^{1/2} \]  

(19)

The material properties of the adhesive AF163-2 and the laminate T40-800/6747-1 were used in Eqn.(18) and Eqn.(19), respectively. It was calculated that, under plane strain conditions, the constant $E_a$ is 1.24 GPa for the cohesive delamination and the constant $E_l$ for the laminate failure is 27.3 GPa for the 0º unidirectional laminate.

Thus, the delamination growth rate of the adhesively-bonded joint can be expressed in Eqn. (20) by combining Eqn.(16) – Eqn.(19).

\[ \frac{da}{dN} = C \left( \frac{\Delta K}{(K_c - K_m)} \right)^b \]  

(20)

where, $C$ and $b$ are material parameters. $K_c$ is the critical value of the stress intensity factor for a given loading mode. $K_m$ is the applied mean stress intensity factor. Using the relationships in Eqn. (18) and Eqn.(19), the R-ratio effect on Mode I delamination growth rate of adhesively-bonded composite joints can be expressed in Eqn.(21).

\[ \frac{da}{dN} = C \left( \frac{\sqrt{\Delta G_1}}{(\sqrt{G_{1c}} - \sqrt{G_{min}})} \right)^b \]  

(21)

where, $\Delta G_1 = \sqrt{G_{1max}} - \sqrt{G_{1min}}$. $C$ and $b$ are material parameters. $G_{1c}$ is the Mode I critical strain energy release rate. $G_{max}$ and $G_{min}$ are the maximum and minimum strain energy release rates, respectively.

Comparison with Experimental Delamination Growth Rate Data

The material parameters in Eqn.(21) based on the life diagram of composite damage accumulation were obtained by nonlinear least square regression. For comparison purposes, the two-parameter models of Hojo [11], Mall [7], and Allegri [14], and a revised three-parameter Allegri model in Eqn.(22) were also applied to fit the experimental results of the Mode I delamination rates for the studied bonded composite joint, as summarized in Table II.

Comparisons between model predictions of delamination growth rate with the experimental data are shown in Fig. 9 to Fig. 12. The model based on the life diagram of composite damage accumulation is characterized by the effective fracture toughness $K_{eff}$ or $G_{eff}$ and a simple power relationship. It can be seen in Fig. 9, the change in slope as a result of the R-ratio effect is well illustrated, and good fit is found for R at 0.1, 0.3 and 0.5. There is a slight discrepancy at 0.7 with the two parameter equation. The model also captures reasonably the observed trend where static delamination rate approached the same value when the maximum energy release rate reaches the critical value ($G_{1max} = G_{1c}$). However, the individual curves of delamination growth rates do not follow the power relationship as illustrated in the test results, leading to an increasing discrepancy between the measured and predicted delamination rate when the growth rate becomes increasingly dominated by cyclic delamination.

As shown in Fig. 10 and Fig. 11, Mall’s and Hojo’s equations are similar in the way that, on the log-log scale, the slopes of all curves with different R-ratio are the same. This type of model is characterized essentially by an
effective facture toughness \(K_{eff} = f(R)\Delta K\). In the log-log scale, the slopes of these curves are the same. Although these models are able to capture the observed power relationship of each curve, but they fail to predict the observed trend wherein static delamination rate approaches the same value when the maximum energy release rate reaches the critical value \((G_{imax} = G_{Ic})\).

While the aforementioned models are all characterized by various definitions of \(K_{eff}\) and a simple power relationship, another way to take into account the effect of \(R\)-ratio is to define it in the exponent of the power relationship such as the Allegri model in Eqn (6). This equation allows for the consideration of 1) a simple power law relationship when \(R\) is a constant; 2) the delamination rate reaches a constant when \(G_{imax} = G_{Ic}\); 3) varying slopes of delamination rates in the log-log scale with the change in \(R\)-ratio. Fig. 12 shows a comparison between the experimental and predicted delamination rate using the Allegri’s model. Good correlations with the experimental data as shown in Fig. 13 were obtained when a revised Allegri’s model in Eqn.(22) was used.

Table II. Summary of Delamination Growth Models for Adhesively-Bonded Composite Joints under Mode I

<table>
<thead>
<tr>
<th>Equation Form</th>
<th>(R^2)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{da}{dN} = 0.00284 \left( \frac{\Delta G_I}{\sqrt{G_{Ic}}} \right)^{5.03} )</td>
<td>0.921</td>
<td>Eqn.(21) (see Fig. 9)</td>
</tr>
<tr>
<td>( \frac{da}{dN} = 0.104 \left( \frac{G_{imax} - G_{Imin}}{G_{Ic}} \right)^{4.51} )</td>
<td>0.893</td>
<td>Mall [9] (see Fig. 10)</td>
</tr>
<tr>
<td>( \frac{da}{dN} = 0.152 \left( \sqrt{G_I} G_r^{0.278} G_{imax}^{0.722} \right)^{9.16} )</td>
<td>0.870</td>
<td>Hojo [11] (see Fig. 11)</td>
</tr>
<tr>
<td>( \frac{da}{dN} = 0.0502 \left( \frac{G_{imax}}{G_{Ic}} \right)^{3.03} )</td>
<td>0.919</td>
<td>Allegri [14] (see Fig. 12)</td>
</tr>
<tr>
<td>( \frac{da}{dN} = 0.0553 \left( \frac{G_{imax}}{G_{Ic}} \right)^{5.58} )</td>
<td>0.940</td>
<td>Eqn.(22) (see Fig. 13)</td>
</tr>
</tbody>
</table>
Fatigue delamination growth of an adhesively-bonded composite joint under Mode I was investigated experimentally and analytically. An automated delamination monitoring system was developed to capture images of delamination tip locations at any cycle in synchronization with peak displacement signals from the load frame. This experimental approach enabled reliable delamination length measurements in a continuous, uninterrupted manner. The range of $R$-ratio for delamination growth tests under the displacement control included 0.1, 0.3, 0.5 and 0.7. The test results showed that no fatigue threshold in the so-called “subcritical region” was observed for the range of tests, and the steady growth region, where the delamination growth of the adhesively-bonded composite joints can be characterized by a simple power law relationship, covered up to $G_{max} = G_{ic}$ for all the studied $R$-ratios under Mode I loading.

Conclusion
An empirical delamination growth model was developed based on a direct association between the damage accumulation rate in the damage zone ahead of the delamination tip and delamination rate. Both the effect of static (or monolithic) and cyclic loads on the damage accumulation in damage zone were considered by using a linear constant life diagram, where the effective stress can be expressed as a function of static and cyclic stress as well as $R$-ratio ($\sigma_{eff} = \Delta\sigma / (\sigma_c - \sigma_{mean})$). It was assumed that a similar relationship for composites under in-plane loading can be applied to composites under out-of-plane loading, provided that the failure mode is matrix or adhesive dominated. A model was then developed based on a simple power relationship. Comparison between the experimental data with the developed model and several others were conducted. It was shown that among the models characterized by $\sigma_{eff}$ and a simple power relationship, the model developed in this work provided the best fit. Another type of delamination growth model characterized by $R$-ratio defined in the power exponent may provide an improved prediction in that it is able to capture all observed trends in the experimental data.

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References


